Comprehensive Notes

of

Central Board of Secondary Education

Class IX

Mathematics

(Ganita Manjari)

Chapter -1 (The use of Coordinates)

by

lumens

Coordinate Geometry:

Coordinate Geometry is a branch of mathematics that helps us study shapes, points, and lines using numbers and graphs.

It combines:

- Geometry → shapes and figures
- Algebra → numbers and equations

In coordinate geometry, every point is represented by a pair of numbers called coordinates.

Example:

- Point A (3, 2)
- Here:
  - 3 → horizontal position
  - 2 → vertical position

Coordinate geometry helps us:

- Locate positions
- Draw graphs
- Measure distance
- Study shapes and patterns

Cartesian Plane:

The graph used in coordinate geometry is called the Cartesian Plane.

It was developed by René Descartes.

Structure of Cartesian Plane:

The plane is formed by two number lines:

X-axis

- Horizontal line
- Positive direction → right side
- Negative direction → left side

Y-axis

- Vertical line
- Positive direction → upward
- Negative direction → downward

These two axes intersect at a point called the Origin.

Origin:

The point where the x-axis and y-axis meet is called the Origin.

Coordinates of origin:

$$(0, 0)$$

- x-coordinate = 0
- y-coordinate = 0

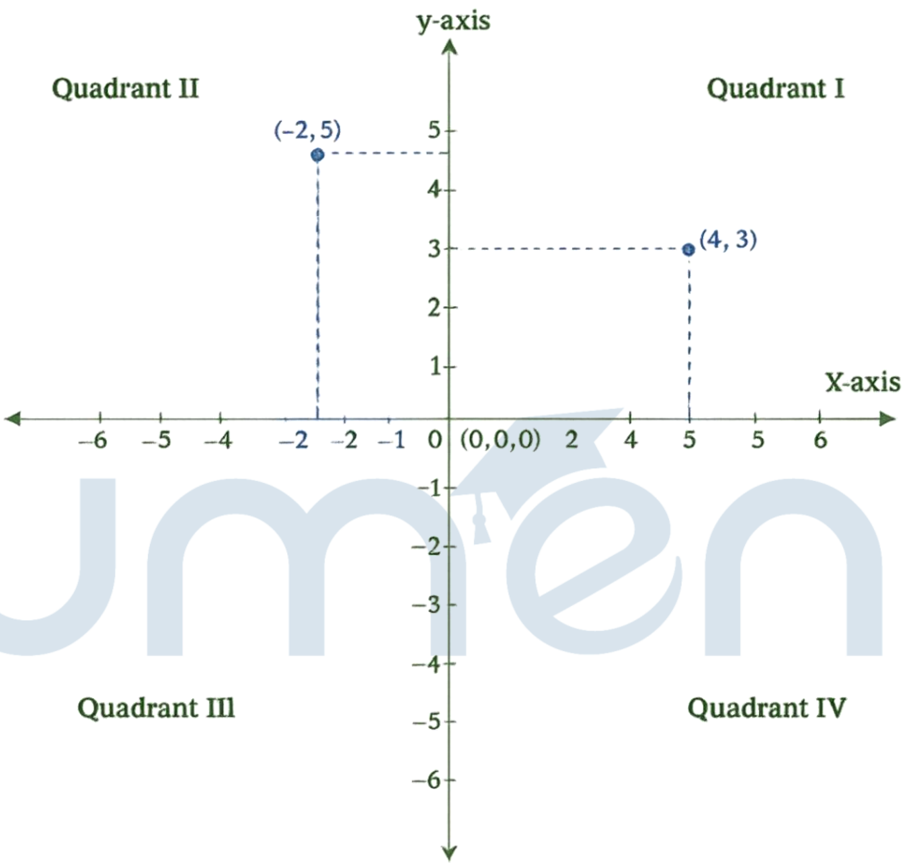
Coordinates of a Point:

A point is represented as: (x, y)

Where:

- x= x-coordinate (horizontal movement)
- y= y-coordinate (vertical movement)

How to Read Coordinates



Example 1

Point: (4, 3)

Meaning:

- Move 4 units right
- Move 3 units up

Example 2

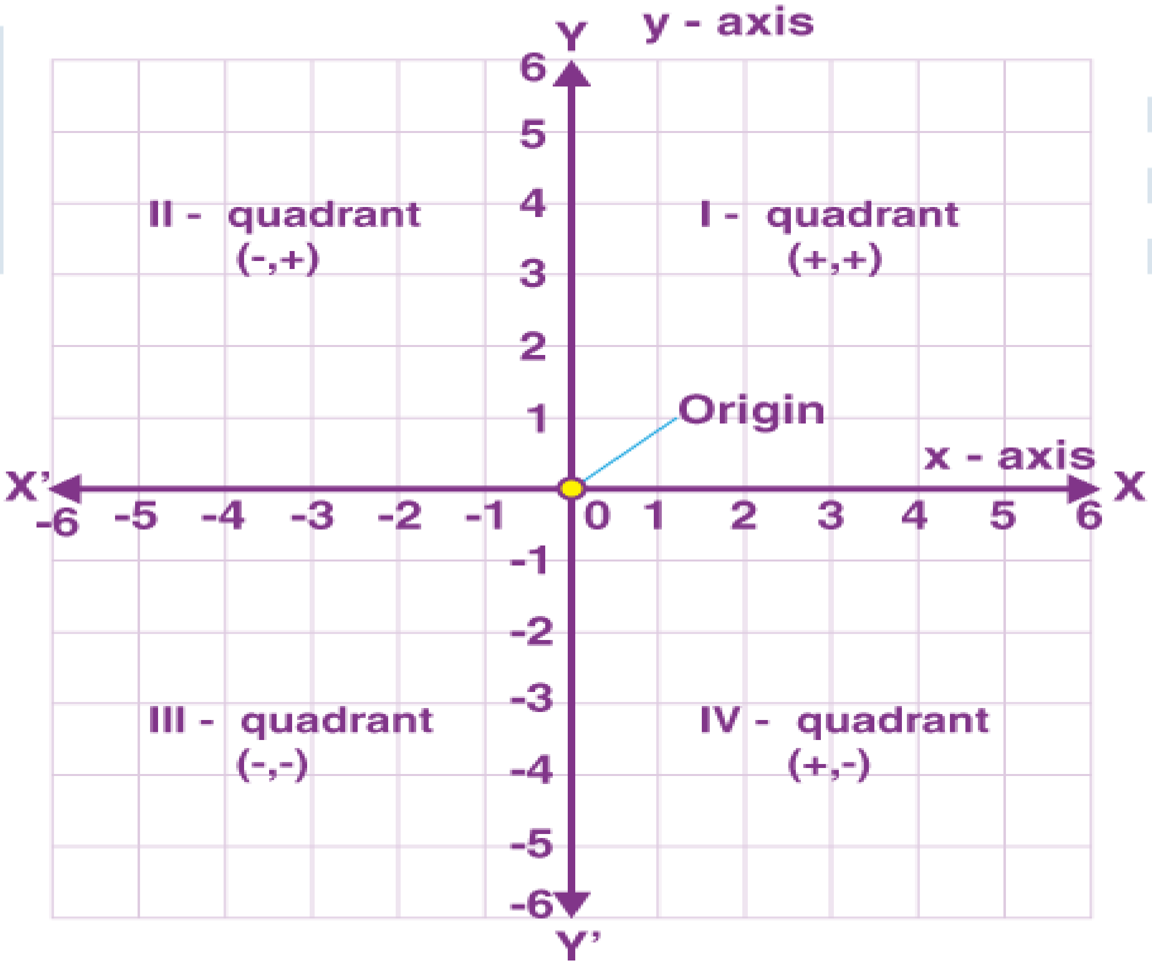
Point: $(-2,5)$

Meaning:

- Move 2 units left
- Move 5 units up

Quadrants:

The x-axis and y-axis divide the plane into four parts called Quadrants.



Names of Quadrants:

First Quadrant (I)

- x positive
- y positive

Example: $(3, 4)$

Second Quadrant (II)

- x negative
- y positive

Example: $(-3,4)$

Third Quadrant (III)

- x negative
- y negative

Example: $(-3, -4)$

Fourth Quadrant (IV)

- x positive
- y negative

Example: $(3, -4)$

Abscissa and Ordinate:

Abscissa:

The x-coordinate is called Abscissa.

Ordinate:

The y-coordinate is called Ordinate.

Example:

For point:

$$(6, -2)$$

- Abscissa $= 6$
- Ordinate $= -2$

Applications of Coordinate Geometry:

Coordinate geometry is used in:

- Maps
- GPS systems
- Video games
- Engineering
- Architecture
- Computer graphics
- Physics
- Robotics

2D Cartesian Coordinate System:

Introduction:

A 2D Cartesian Coordinate System is a method used to locate points on a flat surface using two numbers.

The term:

- 2D means two-dimensional:
    - length
    - width

A Cartesian Coordinate System is formed by:

- a horizontal number line
- a vertical number line

These two lines intersect at a point called the Origin.

The system helps us:

- locate points
- draw graphs
- study shapes and patterns
- understand geometry algebraically

Components of the 2D Cartesian Plane:

The Cartesian Plane consists of:

(a) X-axis:

- Horizontal axis
- Positive direction → right side
- Negative direction → left side

Example:

$$-4, -3, -2, -1, 0, 1, 2, 3, 4$$

(b) Y-axis:

- Vertical axis
- Positive direction → upward
- Negative direction → downward

Example:

$$-4, -3, -2, -1, 0, 1, 2, 3, 4$$

(c) Origin:

The point where both axes intersect is called the Origin.

Coordinates of origin:

$$(0, 0)$$

At origin:

- x-coordinate = 0
- y-coordinate = 0

Points on Axes:

Point on X-axis

If a point lies on x-axis:

$$y = 0$$

Examples:

- (4,0)
- (-6,0)

Point on Y-axis:

If a point lies on y-axis:

$$x = 0$$

Examples:

- (0,5)
- (0,-8)

Reflection of Points in Coordinate Geometry:

Introduction:

In coordinate geometry, reflection means creating a mirror image of a point or figure across a line.

The reflected image:

- has the same shape and size

- remains at equal distance from the mirror line
- appears on the opposite side of the mirror line

The mirror line is called the Line of Reflection.

Real-Life Examples of Reflection:

Reflection is seen in:

- mirrors
- water reflections
- glass surfaces
- symmetry in art and architecture
- computer graphics and animations

Reflection in Coordinate Geometry:

In coordinate geometry, points are reflected across:

1. x-axis
2. y-axis
3. origin
4. line $y = x$
5. line $y = -x$

1. Reflection in the X-axis

When a point is reflected in the x-axis:

- x-coordinate remains same

- y-coordinate changes sign

Rule

$$(x, y) \rightarrow (x, -y)$$

Understanding the Rule

Suppose point:

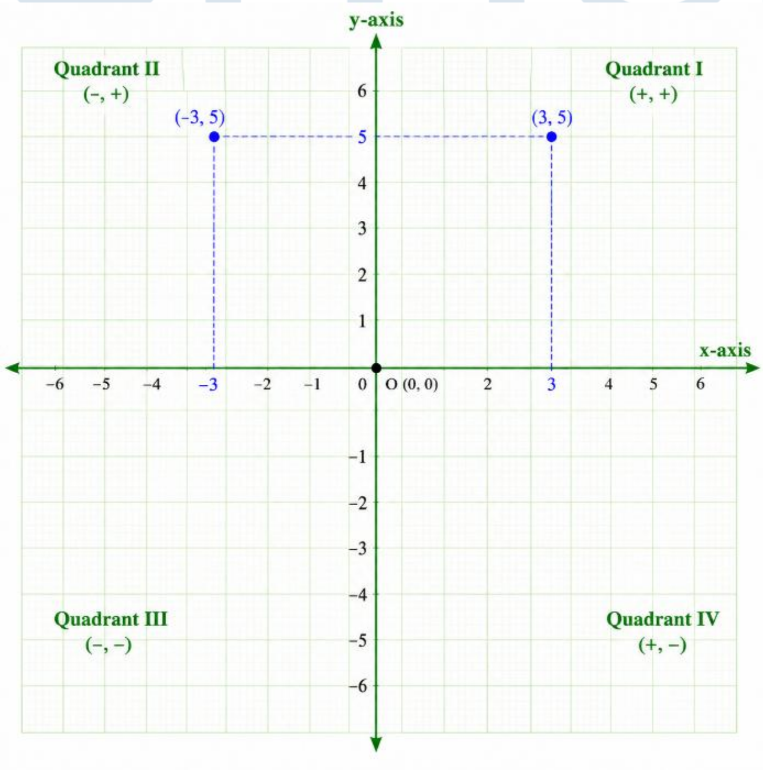
$$(3, 5)$$

After reflection in x-axis:

$$(3, -5)$$

Explanation:

- x remains 3
- y changes from +5 to −5



Example 1

Reflect: $(4,7)$ in x-axis.

Solution

Using rule:

$$(x, y) \to (x, -y)$$
$$(4,7) \to (4, -7)$$

Example 2

Reflect: $(-2, -6)$ in x-axis.

Solution

$$(-2, -6) \to (-2,6)$$

---

2. Reflection in the Y-axis:

When a point is reflected in y-axis:

- y-coordinate remains same
- x-coordinate changes sign

Rule

$$(x, y) \to (-x, y)$$

Example 1

Reflect:

$$(5,3)$$

in y-axis.

Solution

$$(5,3) \to (-5,3)$$

Example 2

Reflect:

$$(-4,7)$$

in y-axis.

Solution

$$(-4,7) \to (4,7)$$

---

3. Reflection in the Origin:

When a point is reflected in origin:

- both coordinates change sign

Rule

$$(x, y) \to (-x, -y)$$

Example 1

Reflect:

$$(3, 4)$$

in origin.

Solution

$$(3,4) \rightarrow (-3,-4)$$

Example 2

Reflect:

$$(-5,2)$$

in origin.

Solution

$$(-5,2) \rightarrow (5,-2)$$

---

4. Reflection in the Line $y = x$

When reflected in line $y = x$:

- x and y coordinates interchange

Rule

$$(x, y) \rightarrow (y, x)$$

Example 1

Reflect:

$$(2, 5)$$

in line $y = x$.

Solution

$$(2,5) \rightarrow (5,2)$$

Example 2

Reflect:

$$(-3,7)$$

in line $y = x$.

Solution

$$(-3,7) \rightarrow (7,-3)$$

---

5. Reflection in the Line $y = -x$

When reflected in line $y = -x$:

- x and y interchange
- both signs change

Rule

$$(x,y) \rightarrow (-y,-x)$$

Example 1

Reflect: $(2, 5)$ in line $y = -x$.

Solution

$$(2,5) \rightarrow (-5,-2)$$

Example 2

Reflect: $(-3,4)$ in line $y = -x$.

Solution

$$(-3,4) \to (-4,3)$$

Summary Table of Reflection Rules:

Reflection	Rule
In x-axis	$(x, y) \to (x, -y)$
In y-axis	$(x, y) \to (-x, y)$
In origin	$(x, y) \to (-x, -y)$
In line $y = x$	$(x, y) \to (y, x)$
In line $y = -x$	$(x, y) \to (-y, -x)$

Reflection of Shapes:

Reflection can also be applied to:

- triangles
- squares
- polygons
- graphs

Each vertex is reflected separately.

Example: Reflection of Triangle

Suppose triangle has vertices:

$$A(1,2),\ B(3,4),\ C(5,1)$$

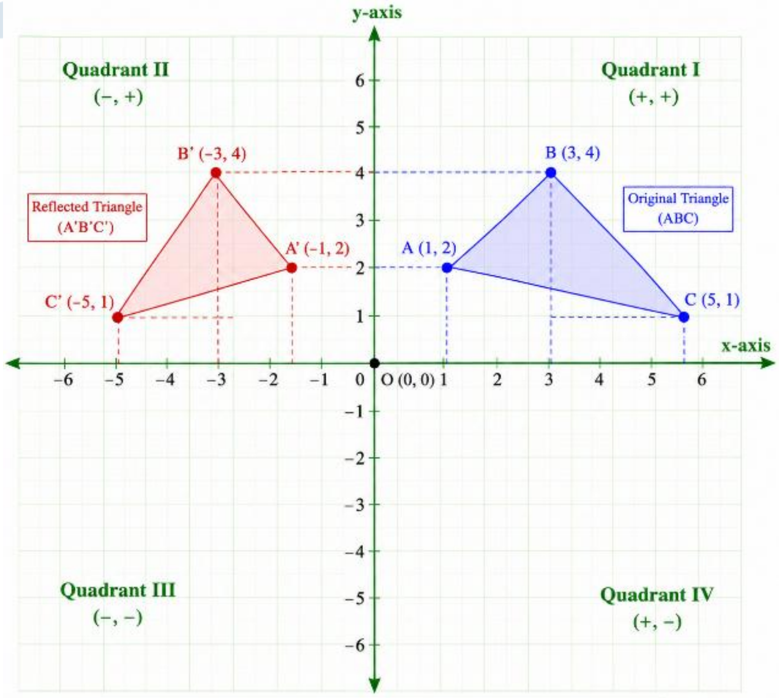
Reflect in x-axis.

Using:

$$(x, y) \rightarrow (x, -y)$$

New vertices:

$$A'(1, -2)$$
$$B'(3, -4)$$
$$C'(5, -1)$$



The triangle ABC is reflected across the y-axis to form triangle A'B'C'.
Reflection rule: $(x, y) \rightarrow (-x, y)$

Properties of Reflection:

1. Distance Remains Same:

Original point and reflected point are equally distant from mirror line.

2. Shape and Size Remain Same:

Reflection changes orientation but not dimensions.

3. Reflection Produces Symmetry:

Reflected figure becomes mirror image.

Solved Examples:

Example 1

Find reflection of:

$$(7, -3)$$

in x-axis.

Solution

Rule:

$$(x, y) \rightarrow (x, -y)$$
$$(7, -3) \rightarrow (7,3)$$

Example 2

Find reflection of:

$$(-6,2)$$

in y-axis.

Solution

$$(-6,2) \rightarrow (6,2)$$

Example 3

Find reflection of:

$$(4,-5)$$

in origin.

Solution

$$(4,-5) \rightarrow (-4,5)$$

Example 4

Find reflection of:

$$(3,8)$$

in line $y = x$.

Solution

$$(3,8) \rightarrow (8,3)$$

Distance Formula:

Suppose there are two points:

$$A(x_1, y_1)$$

and

$$B(x_2, y_2)$$

The distance between these two points is represented by:

$$AB$$

The formula used to find this distance is called the Distance Formula.

Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where:

- d= distance between two points
- (x_1, y_1)= coordinates of first point
- (x_2, y_2)= coordinates of second point

Understanding the Idea:

Suppose:

$$A(x_1, y_1)$$

and

$B(x_2, y_2)$

If we draw horizontal and vertical lines between the points, a right triangle is formed.

Horizontal Distance: $x_2 - x_1$

Vertical Distance: $y_2 - y_1$

These become the two perpendicular sides of a right triangle.

The actual distance between the points becomes the hypotenuse.

Using Pythagorean theorem:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Taking square root:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Important Terms:

Term	Meaning
Coordinates	Position of a point
Ordered Pair	Point written as (x, y)
Horizontal Distance	Difference of x-values
Vertical Distance	Difference of y-values
Distance	Length between two points

Steps to Use Distance Formula:

Step 1

Write coordinates of both points.

Step 2

Identify:

- x_1, y_1

- x_2, y_2

Step 3

Substitute values into formula.

Step 4

Simplify carefully.

Step 5

Find square root.

Solved Examples:

Example 1

Find the distance between: (1,2) and (4,6)

Solution: Using distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

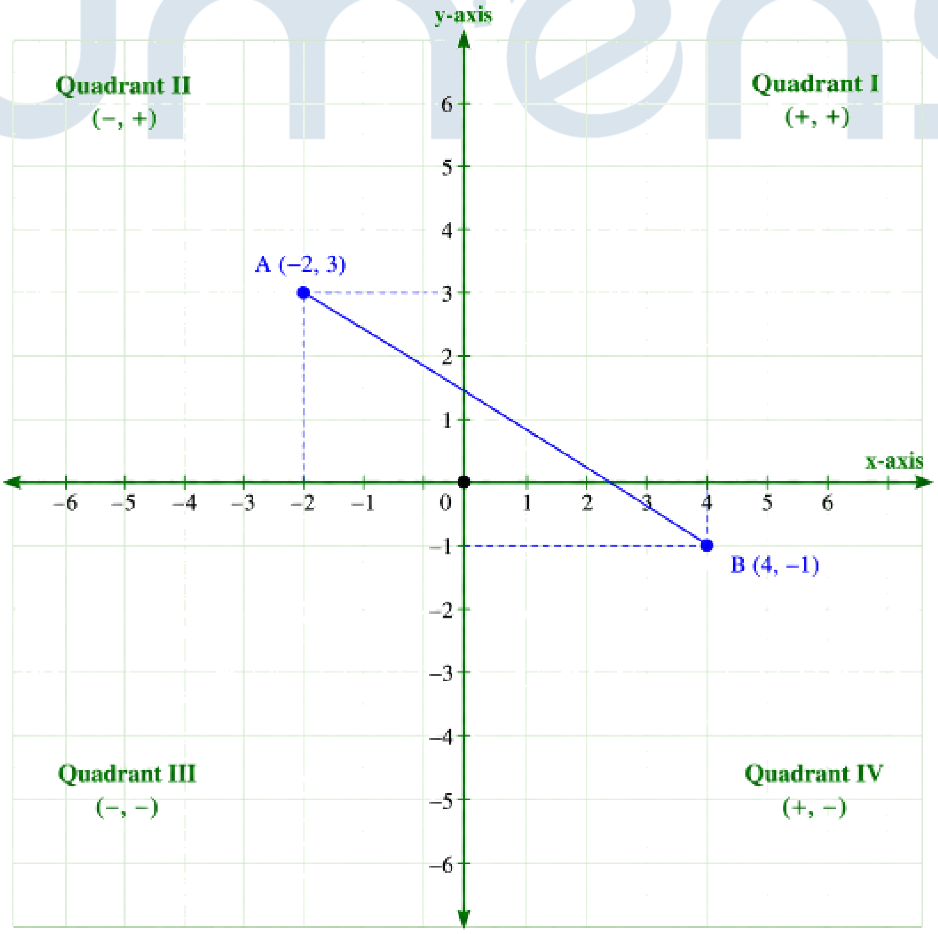
Substitute values: $$d = \sqrt{(4-1)^2 + (6-2)^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5$$

Answer : Distance = 5 units

Example 2

Find distance between:

$$(-2,3) \text{ and } (4,-1)$$



Solution

$$d = \sqrt{(4+2\,)^2 + (-1-3)^2}$$
$$= \sqrt{6^2 + (-4)^2}$$
$$= \sqrt{36+16}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$

Answer: $2\sqrt{13}$ units

Example 3

Find distance between:

$$(0,0) \text{ and } (5,12)$$

Solution

$$d = \sqrt{(5-0)^2 + (12-0)^2}$$
$$= \sqrt{25+144}$$
$$= \sqrt{169}$$
$$= 13$$

Answer: Distance = 13 units

Example 4

Find distance between:

$$(-3,-4) \text{ and } (3,4)$$

Solution

$$d = \sqrt{()^2 + (4 - (-4))^2}$$
$$= \sqrt{6^2 + 8^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100}$$
$$= 10$$

Answer: Distance $= 10$ units

Special Cases of Distance Formula:

(a) Horizontal Distance:

If two points have same y-coordinate:

$$y_1 = y_2$$

then distance becomes:

$$d = | x_2 - x_1 |$$

Example

Find distance between:

(2,5) and (9,5)

Solution

$$d = | 9 - 2 |$$
$$= 7$$

Answer $= 7$ units

(b) Vertical Distance

If two points have same x-coordinate:

$$x_1 = x_2$$

then distance becomes:

$$d = | y_2 - y_1 |$$

Example

Find distance between:

(4,2) and (4,10)

Solution

$$d = | 10 - 2 |$$
$$= 8$$

Answer = 8 units

Applications of Distance Formula:

(a) GPS Navigation: Used for measuring shortest routes.

(b) Engineering: Used in construction and machine design.

(c) Computer Graphics: Used in animations and game development.

(d) Robotics: Used for movement calculations.

(e) Physics: Used in motion and displacement calculations.

Solved Mixed Examples:

Example 1

Find the distance between:

$$(7,3) \text{ and } (1,-5)$$

Solution

$$d = \sqrt{(1-7)^2 + (-5-3)^2}$$
$$= \sqrt{(-6)^2 + (-8)^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100}$$
$$= 10$$

Answer $= 10$ units

Example 2

Find the value of k if distance between:

$$(2,3) \text{ and } (2,k)$$

is 7 units.

Solution

Since x-coordinates are same:

$$d = |k-3|$$

Given:

$$|k - 3| = 7$$

Case 1:

$$k - 3 = 7$$
$$k = 10$$

Case 2:

$$k - 3 = -7$$
$$k = -4$$

Answer

$$k = 10 \text{ or } -4$$

Practice Questions:

A. Find the Distance

1. Between $(1, 2)$ and $(4, 6)$
2. Between $(0, 0)$ and $(8, 15)$
3. Between $(-2,5)$ and $(4, -3)$
4. Between $(6, 7)$ and $(6, -1)$
5. Between $(3, 4)$ and $(9, 4)$

B. Fill in the Blanks

1. Distance formula is based on _______ theorem.
2. Distance between same points is _______.

3. Distance can never be _______.
4. If y-coordinates are same, distance is called _______ distance.
5. Straight line distance is the _______ distance between two points.

C. True / False

1. Distance formula uses square root.
2. Distance can be negative.
3. Distance between (0, 0)and (3, 4)is 5.
4. Horizontal distance uses difference of y-values.
5. Distance formula is based on Pythagorean theorem.

D. Multiple Choice Questions

1. Distance between (0, 0)and (3, 4)is:

A. 4
B. 5
C. 6
D. 7

2. Distance formula is based on:

A. Reflection rule
B. Midpoint formula
C. Pythagorean theorem
D. Algebraic identities

3. Distance between same points is:

A. 1
B. –1
C. 0
D. undefined

4. Which formula finds shortest distance?

A. Midpoint formula
B. Reflection formula
C. Distance formula
D. Quadrant rule

Answer Key:

A. Distance

1. 5
2. 17
3. 10
4. 8
5. 6

B. Fill in the Blanks

1. Pythagorean
2. zero
3. negative
4. horizontal

5. shortest

C. True / False

1. True
2. False
3. True
4. False
5. True

D. MCQs

1. B
2. C
3. C
4. C

Midpoint Formula in Coordinate Geometry

Introduction:

In coordinate geometry, the Midpoint Formula is used to find the exact middle point between two points.

The midpoint divides a line segment into:

- two equal parts
- equal distances from both endpoints

The midpoint formula is very useful in:

- geometry

- graphing
- engineering
- architecture
- computer graphics
- design and symmetry

What is a Midpoint?

Suppose two points are given:

$$A(x_1, y_1)$$

and

$$B(x_2, y_2)$$

The point exactly halfway between them is called the Midpoint.

It is usually represented by:

$$M$$

Midpoint Formula:

The midpoint of points:

$$A(x_1, y_1)$$

and

$$B(x_2, y_2)$$

is given by:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Understanding the Formula:

The midpoint is found by:

- taking average of x-coordinates
- taking average of y-coordinates

That means:

x-coordinate of midpoint

$$\frac{x_1 + x_2}{2}$$

y-coordinate of midpoint

$$\frac{y_1 + y_2}{2}$$

Important Terms

Term	Meaning
Coordinates	Position of a point
Ordered Pair	Point written as (x, y)
Midpoint	Exact center between two points
Line Segment	Straight line joining two points

Steps to Find Midpoint:

Step 1

Write coordinates of both points.

Step 2

Identify:

- x_1, y_1
- x_2, y_2

Step 3

Substitute values into midpoint formula.

Step 4

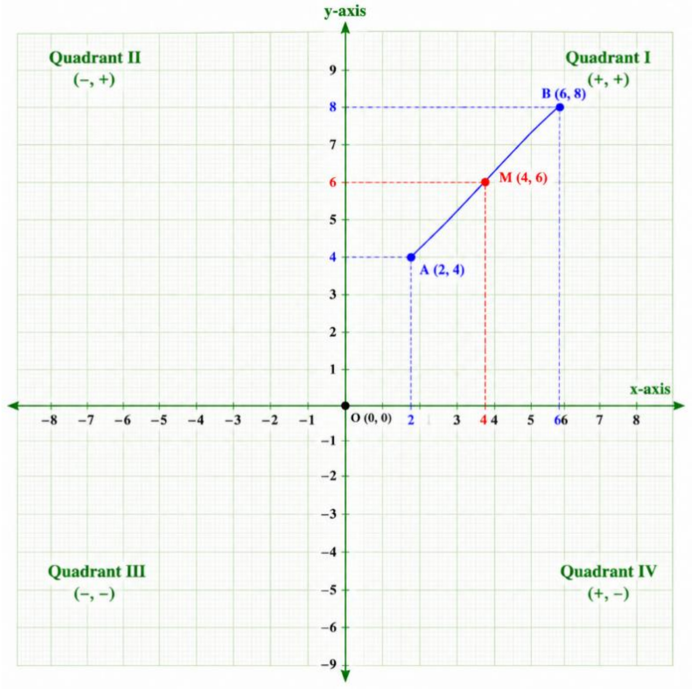
Simplify carefully.

Solved Examples

Example 1

Find midpoint of:

$$(2,4) \text{ and } (6,8)$$



Solution

Using midpoint formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute values:

$$\left(\frac{2+6}{2}, \frac{4+8}{2}\right)$$
$$= \left(\frac{8}{2}, \frac{12}{2}\right)$$
$$= (4,6)$$

Answer

Midpoint $= (4, 6)$

Example 2

Find midpoint of:

$$(-3,5) \text{ and } (7,-1)$$

Solution

$$\left(\frac{-3+7}{2},\frac{5+(-1)}{2}\right)$$
$$=\left(\frac{4}{2},\frac{4}{2}\right)$$
$$=(2,2)$$

Answer: Midpoint $=(2,2)$

Example 3

Find midpoint of:

$$(0,0) \text{ and } (8,10)$$

Solution

$$\left(\frac{0+8}{2},\frac{0+10}{2}\right)$$
$$=(4,5)$$

Answer: Midpoint $=(4,5)$

Example 4

Find midpoint of:

$$(-6,-4) \text{ and } (2,8)$$

Solution

$$\left(\frac{-6+2}{2},\frac{-4+8}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{4}{2}\right)$$
$$= (-2,2)$$

Answer: Midpoint $= (-2,2)$

Example 5

Find midpoint of:

$$(5,-3) \text{ and } (9,7)$$

Solution

$$\left(\frac{5+9}{2}, \frac{-3+7}{2}\right)$$
$$= \left(\frac{14}{2}, \frac{4}{2}\right)$$
$$= (7,2)$$

Answer

Midpoint $= (7, 2)$

Geometrical Meaning of Midpoint:

The midpoint:

- lies exactly halfway
- divides line segment into equal parts
- is equally distant from both endpoints

If:

$$M$$

is midpoint of:

$$AB$$

then:

$$AM = MB$$

Special Cases:

(a) Midpoint on X-axis

If midpoint lies on x-axis:

$$y = 0$$

Example

Find midpoint of:

$$(2,4) \text{ and } (6,-4)$$

Solution

$$\left(\frac{2+6}{2}, \frac{4+(-4)}{2}\right)$$
$$= (4,0)$$

Midpoint lies on x-axis.

(b) Midpoint on Y-axis

If midpoint lies on y-axis:

$$x = 0$$

Example

Find midpoint of:

$$(-5,3) \text{ and } (5,7)$$

Solution

$$\left(\frac{-5+5}{2}, \frac{3+7}{2}\right)$$
$$= (0,5)$$

Midpoint lies on y-axis.

Finding Missing Endpoint Using Midpoint Formula

Sometimes midpoint and one endpoint are given.

We can find the missing point.

Example

Midpoint of: $A(2,4)$ and $B(x,8)$ is $(5,6)$

Find x.

Solution

Using midpoint formula:

$$\frac{2+x}{2} = 5$$

Multiply by 2:

$$2 + x = 10$$
$$x = 8$$

Answer

$$x = 8$$

Applications of Midpoint Formula:

(a) Architecture: Used in finding centers and symmetry.

(b) Engineering: Used in design and measurement.

(c) Computer Graphics: Used in animations and object positioning.

(d) Geometry: Used in triangles, polygons, and constructions.

(e) Navigation: Used in locating halfway positions.

Comparison Between Distance Formula and Midpoint Formula:

Distance Formula	Midpoint Formula
Finds length	Finds center point
Gives numerical answer	Gives coordinates
Uses square root	Uses averages

Mixed Solved Example:

Points are:

$$A(1,3), B(7,11)$$

Find midpoint.

Solution

Using midpoint formula:

$$\left(\frac{1+7}{2}, \frac{3+11}{2}\right)$$
$$= \left(\frac{8}{2}, \frac{14}{2}\right)$$
$$= (4,7)$$

Answer

Midpoint $= (4,7)$

Practice Questions:

A. Find the Midpoint

1. Between $(2, 6)$ and $(8, 10)$
2. Between $(-4,2)$ and $(6, 8)$
3. Between $(0, 0)$ and $(10, 12)$
4. Between $(3, -5)$ and $(7, 9)$
5. Between $(-2, -4)$ and $(4, 6)$

B. Find Missing Coordinate

1. Midpoint of $(2, 4)$ and $(x, 8)$ is $(5, 6)$. Find x.
2. Midpoint of $(3, y)$ and $(7, 9)$ is $(5, 6)$. Find y.

C. Fill in the Blanks

1. Midpoint divides a line segment into _______ equal parts.
2. Midpoint formula uses _______ of coordinates.
3. Midpoint gives coordinates of the _______ point.
4. Midpoint lies exactly _______ between two points.
5. Midpoint of identical points is the _______ point itself.

D. True / False

1. Midpoint formula finds length of line segment.
2. Midpoint divides line into equal parts.
3. Midpoint formula uses averages.
4. Midpoint always lies outside the segment.

5. Midpoint gives coordinates.

E. Multiple Choice Questions

1. Midpoint of (2, 4)and (6, 8)is:

A. (3, 5)
B. (4, 6)
C. (5, 7)
D. (6, 8)

2. Midpoint formula uses:

A. multiplication
B. square root
C. averages
D. subtraction only

3. Midpoint of (0, 0)and (8, 10)is:

A. (4, 5)
B. (5, 4)
C. (8, 10)
D. (0, 0)

4. Midpoint gives:

A. distance
B. slope
C. center point
D. angle

Answer Key:

A. Midpoints

1. (5, 8)
2. (1, 5)
3. (5, 6)
4. (5, 2)
5. (1, 1)

B. Missing Coordinates

1. $x = 8$
2. $y = 3$

C. Fill in the Blanks

1. two
2. averages
3. center
4. halfway
5. same

D. True / False

1. False
2. True
3. True
4. False

5. True

E. MCQs

1. B
2. C
3. A
4. C

Collinearity of Three Points in Coordinate Geometry

Introduction

In coordinate geometry, points are said to be collinear if they lie on the same straight line.

The concept of collinearity is very important in:

- geometry
- graphing
- engineering
- architecture
- computer graphics
- map design

Understanding collinearity helps us:

- determine whether points form a straight line
- study shapes and polygons
- solve geometrical problems

What are Collinear Points?

Three or more points lying on the same straight line are called Collinear Points.

Example:

$$(1,1),\ (2,2),\ (3,3)$$

These points lie on one straight line, so they are collinear.

Non-Collinear Points

Points that do not lie on the same straight line are called Non-Collinear Points.

Example:

$$(1,2),\ (3,4),\ (5,1)$$

These do not form one straight line.

Methods to Check Collinearity:

We can check collinearity using:

1. Distance Method
2. Slope Method
3. Area of Triangle Method

Method 1 : Distance Method:

Suppose three points are:

$$A,\ B,\ C$$

If:

$$AB + BC = AC$$

then the points are collinear.

This means:

- one point lies between the other two
- all three points form a straight line

Distance Formula

To find distances, we use:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solved Example 1 (Distance Method)

Check whether:

$$(1,1),\ (2,2),\ (3,3)$$

are collinear.

Step 1 : Find AB

$$AB = \sqrt{(2-1)^2 + (2-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2}$$

Step 2 : Find BC

$$BC = \sqrt{(3-2)^2 + (3-2)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2}$$

Step 3 : Find AC

$$AC = \sqrt{(3-1)^2 + (3-1)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

Step 4 : Compare

$$AB + BC = \sqrt{2} + \sqrt{2}$$
$$= 2\sqrt{2}$$
$$= AC$$

Hence the points are collinear.

Method 2 : Slope Method

If slopes between pairs of points are equal, then points are collinear.

Slope Formula

The slope between two points is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Condition for Collinearity:

If:

$$m_{AB} = m_{BC}$$

then points are collinear.

Solved Example 2 (Slope Method)

Check whether:

$$(1,2),\ (3,6),\ (5,10)$$

are collinear.

Step 1 : Find slope of AB

$$m_{AB} = \frac{6-2}{3-1}$$
$$= \frac{4}{2}$$
$$= 2$$

Step 2 : Find slope of BC

$$m_{BC} = \frac{10-6}{5-3}$$
$$= \frac{4}{2}$$

$$= 2$$

Step 3 : Compare Slopes

$$m_{AB} = m_{BC}$$

Hence the points are collinear.

Solved Example 3

Check whether:

$$(1,2),\ (3,4),\ (5,8)$$

are collinear.

Step 1 : Find slope of AB

$$m_{AB} = \frac{4-2}{3-1}$$
$$= \frac{2}{2}$$
$$= 1$$

Step 2 : Find slope of BC

$$m_{BC} = \frac{8-4}{5-3}$$
$$= \frac{4}{2}$$
$$= 2$$

Step 3 : Compare

$$1 \neq 2$$

Hence the points are not collinear.

Method 3 : Area of Triangle Method:

If three points are collinear, then the triangle formed by them has zero area.

Area Formula:

Area of triangle formed by:

$$(x_1, y_1),\ (x_2, y_2),\ (x_3, y_3)$$

is:

$$\frac{1}{2} \mid x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \mid$$

Condition for Collinearity

If:

$$\text{Area} = 0$$

then points are collinear.

Solved Example 4 (Area Method)

Check whether:

$$(1,1),\ (2,2),\ (3,3)$$

are collinear.

Solution

Using area formula:

$$\frac{1}{2} \mid 1(2-3)+2(3-1)+3(1-2) \mid$$
$$= \frac{1}{2} \mid -1+4-3 \mid$$
$$= \frac{1}{2}(0)$$
$$= 0$$

Hence points are collinear.

Important Observations

(a) Collinear Points Form Straight Line:

They always lie on one straight line.

(b) Non-Collinear Points Form Triangle:

If points are not collinear, they form a triangle.

(c) Equal Slopes Mean Straight Line:

Same slope indicates same direction.

Real-Life Applications:

(a) Road Design

Roads and railway tracks use straight-line alignment.

(b) Architecture

Used in alignment of walls and structures.

(c) Computer Graphics

Used in line drawing and object positioning.

(d) Navigation

Used in path calculations.

Solved Mixed Examples

Example 5

Check whether:

$$(-1,-2),\ (0,1),\ (1,4)$$

are collinear.

Step 1 : Find slope of first pair

$$m_1 = \frac{1-(-2)}{0-(-1)}$$

$$= \frac{3}{1}$$
$$= 3$$

Step 2 : Find slope of second pair

$$m_2 = \frac{4-1}{1-0}$$
$$= \frac{3}{1}$$
$$= 3$$

Step 3 : Compare

$$m_1 = m_2$$

Hence points are collinear.

Example 6

Check whether:

$$(2,3),\ (4,7),\ (6,10)$$

are collinear.

Step 1 : Find slopes

$$m_1 = \frac{7-3}{4-2}$$
$$= \frac{4}{2}$$
$$= 2$$

$$m_2 = \frac{10-7}{6-4}$$
$$= \frac{3}{2}$$

Step 2 : Compare

$$2 \neq \frac{3}{2}$$

Hence points are not collinear.

Summary Table:

Method	Condition
Distance Method	$AB + BC = AC$
Slope Method	Equal slopes
Area Method	Area = 0

Practice Questions

A. Check Whether Collinear

1. $(1,1),\ (2,2),\ (3,3)$
2. $(2,3),\ (4,5),\ (6,7)$
3. $(1,2),\ (3,4),\ (5,8)$
4. $(-1,-2),\ (0,1),\ (1,4)$
5. $(2,1),\ (4,3),\ (6,6)$

B. Fill in the Blanks

1. Points on the same straight line are called _______ points.
2. Equal _______ indicate collinearity.
3. Area of triangle formed by collinear points is _______.
4. Collinear points form a _______ line.
5. Distance method uses the _______ formula.

C. True / False

1. Collinear points lie on same line.
2. Non-collinear points always form a triangle.
3. Equal slopes indicate collinearity.
4. Area of triangle for collinear points is non-zero.
5. Distance formula helps in checking collinearity.

D. Multiple Choice Questions

1. Collinear points lie on:

A. circle
B. curve
C. straight line
D. triangle

2. Which method uses equal slopes?

A. Distance method
B. Slope method
C. Area method
D. Reflection method

3. Area formed by collinear points is:

A. 1
B. 2
C. 0
D. undefined

4. Which formula is used in slope method?

A. Midpoint formula
B. Reflection formula
C. Slope formula
D. Quadrant formula

Answer Key

A.

1. Collinear
2. Collinear
3. Not collinear
4. Collinear
5. Not collinear
6.

B.

1. collinear
2. slopes
3. zero
4. straight
5. distance
6.

C.

1. True
2. True
3. True
4. False
5. True
6.

D.

1. C
2. B
3. C
4. C

Practice Worksheet

Section A — Multiple Choice Questions

Choose the correct answer.

1. The point where x-axis and y-axis intersect is called:

A. Quadrant
B. Origin
C. Coordinate
D. Axis

2. Coordinates of origin are:

A. $(1,1)$
B. $(0,1)$
C. $(1,0)$
D. $(0,0)$

3. Point $(3,-4)$lies in:

A. First quadrant
B. Second quadrant
C. Third quadrant
D. Fourth quadrant

4. Point $(-5,2)$lies in:

A. I quadrant
B. II quadrant
C. III quadrant
D. IV quadrant

5. Reflection of $(3, 5)$ in x-axis is:

A. $(-3,5)$
B. $(3,-5)$
C. $(-3,-5)$
D. $(5, 3)$

6. Reflection of $(4,-2)$ in y-axis is:

A. $(-4,-2)$
B. $(4, 2)$
C. $(-4,2)$
D. $(2, 4)$

7. Reflection of $(2, 7)$ in origin is:

A. $(-2,-7)$
B. $(2,-7)$
C. $(-2,7)$
D. $(7, 2)$

8. Midpoint of $(2, 4)$ and $(6, 8)$ is:

A. $(3,5)$
B. $(4,6)$
C. $(5,7)$
D. $(6,8)$

9. Distance between $(0, 0)$ and $(3, 4)$ is:

A. 3
B. 4

C. 5
D. 6

10. Which theorem is used in distance formula?

A. Midpoint theorem
B. Pythagorean theorem
C. Reflection theorem
D. Euclid theorem

11. The point (0, 5)lies on:

A. x-axis
B. y-axis
C. origin
D. none

12. If three points lie on the same straight line, they are called:

A. Coplanar
B. Parallel
C. Collinear
D. Perpendicular

13. Area formed by collinear points is:

A. 1
B. 2
C. 0
D. Undefined

14. Reflection in line $y = x$changes:

A. sign only
B. x-coordinate only
C. y-coordinate only
D. x and y interchange

15. Distance between identical points is:

A. 0
B. 1
C. –1
D. Undefined

16. The midpoint divides a line segment into:

A. three equal parts
B. four equal parts
C. two equal parts
D. unequal parts

17. Point $(-3, -5)$lies in:

A. I quadrant
B. II quadrant
C. III quadrant
D. IV quadrant

18. Reflection of $(5, -3)$in x-axis is:

A. (5, 3)
B. $(-5, -3)$

C. $(-5,3)$
D. $(3,5)$

19. Slope of a horizontal line is:

A. 1
B. 0
C. undefined
D. –1

20. Distance formula gives:

A. midpoint
B. angle
C. shortest distance
D. reflection

Section B — Fill in the Blanks

1. The horizontal axis is called the _______ axis.
2. The vertical axis is called the _______ axis.
3. Coordinates are written as _______ pairs.
4. The point $(0,0)$is called _______.
5. Reflection in x-axis changes the sign of _______ coordinate.
6. Reflection in y-axis changes the sign of _______ coordinate.
7. Reflection in origin changes signs of _______ coordinates.
8. The midpoint formula uses _______ of coordinates.
9. Distance formula is based on _______ theorem.

10. Distance can never be _______.
11. Midpoint divides a line segment into _______ equal parts.
12. Three points on same straight line are called _______ points.
13. Area of triangle formed by collinear points is _______.
14. Point $(4, 0)$lies on the _______ axis.
15. Point $(0, -7)$lies on the _______ axis.
16. In second quadrant x-coordinate is _______.
17. In fourth quadrant y-coordinate is _______.
18. Reflection in line $y = x$interchanges _______ and _______.
19. Distance between same points is _______.
20. A straight line is the _______ distance between two points.

Section C — True / False

1. x-axis is horizontal.
2. y-axis is horizontal.
3. Origin is $(0, 0)$.
4. Reflection changes size of figure.
5. Reflection in x-axis changes y-coordinate.
6. Reflection in y-axis changes x-coordinate.
7. Midpoint formula finds distance.
8. Distance formula uses square root.
9. Distance can be negative.
10. Collinear points lie on same line.
11. Non-collinear points form a triangle.

12. Point $(3,-4)$lies in fourth quadrant.

13. Point $(-2,5)$lies in third quadrant.

14. Reflection in origin changes both coordinates.

15. Midpoint lies halfway between two points.

16. Distance between $(0, 0)$and $(3, 4)$is 5.

17. Area formed by collinear points is zero.

18. Point $(0, 6)$lies on x-axis.

19. Reflection in line $y = x$interchanges coordinates.

20. Ordered pairs $(2, 5)$and $(5, 2)$are same.

Section D — Solve Using Distance Formula

1.: Find distance between: $(2,3)$ and $(6,7)$

2. Find distance between: $(-1,4)$ and $(3,-2)$

3. Find distance between: $(0,0)$ and $(8,15)$

4. Find distance between: $(5,2)$ and $(5,10)$

5. Find distance between: $(-3,-4)$ and $(3,4)$

Section E — Solve Using Midpoint Formula

1. Find midpoint of: $(2,4)$ and $(6,8)$

2. Find midpoint of: $(-3,5)$ and $(7,-1)$

3. Find midpoint of: $(0,0)$ and $(10,12)$

4. Find midpoint of: $(3,-5)$ and $(7,9)$

5. Find midpoint of: $(-2,-4)$ and $(4,6)$

Section F — Check Whether Triangle is Right-Angled

1. Check whether triangle formed by: $(0,0)$, $(3,0)$, $(0,4)$ is right-angled.

2. Check whether triangle formed by: $(1,1)$, $(4,5)$, $(7,1)$ is right-angled.

3. Check whether triangle formed by: $(2,3)$, $(6,3)$, $(6,8)$ is right-angled.

Section G — Check Collinearity of Points

1. Check whether: $(1,1)$, $(2,2)$, $(3,3)$ are collinear.

2. Check whether: $(2,3)$, $(4,5)$, $(6,7)$ are collinear.

3. Check whether: $(1,2)$, $(3,4)$, $(5,8)$ are collinear.

Answer Key

Section A — MCQs

1. B
2. D
3. D
4. B
5. B
6. A
7. A
8. B
9. C
10. B
11. B
12. C
13. C
14. D
15. A
16. C
17. C
18. A
19. B
20. C

Section B — Fill in the Blanks

1. x
2. y
3. ordered
4. origin
5. y
6. x
7. both
8. averages
9. Pythagorean
10. negative
11. two
12. collinear
13. zero
14. x
15. y
16. negative
17. negative
18. x, y
19. zero
20. shortest

Section C — True / False

1. True
2. False
3. True
4. False
5. True
6. True
7. False
8. True
9. False
10. True
11. True
12. True
13. False
14. True
15. True
16. True
17. True
18. False
19. True
20. False

Section D — Distance Formula Answers

1. $4\sqrt{2}$
2. $2\sqrt{13}$
3. 17
4. 8
5. 10

Section E — Midpoint Formula Answers

1. (4, 6)
2. (2, 2)
3. (5, 6)

4. (5, 2)
5. (1, 1)

Section F — Right-Angled Triangle Answers

1. Yes
2. No
3. Yes

Section G — Collinearity Answers

1. Collinear
2. Collinear
3. Not Collinear

End of Chapter