Comprehensive Notes

of

Central Board of Secondary Education

Class IX

Mathematics

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Chapter -3 (The world of Numbers)

by

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Kinds of Numbers

Numbers are used in mathematics to count, measure, compare, and solve problems. Numbers are divided into different groups based on their properties.

(a) Natural Numbers:

Natural numbers are counting numbers.

Examples: 1, 2, 3, 4, 5, 6, ...

Properties

- Smallest natural number is 1.
- There is no largest natural number.
- Natural numbers do not include 0 or negative numbers.

(b) Whole Numbers:

Whole numbers include all natural numbers along with 0.

Examples: 0, 1, 2, 3, 4, 5, ...

Properties

- Smallest whole number is 0.
- Whole numbers do not include negative numbers.

(c) Integers:

Integers include positive numbers, negative numbers, and zero.

Examples: ..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

Types of Integers:

1. Positive integers: 1, 2, 3, ...
2. Negative integers: -1, -2, -3, ...
3. Zero

(d) Rational Numbers:

A rational number is any number that can be written in the form:

$$\frac{p}{q}$$

where:

- p and q are integers
- q ≠ 0

Examples

$$\frac{1}{2}, \frac{-3}{5}, 7, 0, 0.25$$

Important Points

- Every integer is a rational number.
- Rational numbers may have terminating or recurring decimals.

(e) Irrational Numbers:

Numbers that cannot be written in the form $\frac{p}{q}$ are called irrational numbers.

Examples:

$$\sqrt{2}, \sqrt{5}, \pi$$

Properties

- Decimal expansion is non-terminating and non-recurring.
- Cannot be expressed as fractions.

(f) Real Numbers:

All rational and irrational numbers together form real numbers.

Examples:

$$-3, \frac{2}{5}, \sqrt{7}, \pi$$

Equivalent Rational Numbers

Two rational numbers are called equivalent if they represent the same value.

Example 1

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$$

All these fractions represent the same point on the number line.

How to Find Equivalent Rational Numbers

Multiply or divide numerator and denominator by the same non-zero number.

Example 2

Find three equivalent rational numbers of $\frac{3}{5}$.

Multiply numerator and denominator by 2, 3, and 4.

$$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20}$$

Addition of Rational Numbers

Case 1: Same Denominator

Add the numerators and keep the denominator same.

Example

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$$

Case 2: Different Denominators

Find LCM of denominators.

Example

$$\frac{1}{2} + \frac{1}{3}$$

LCM of 2 and 3 is 6.

$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{1}{3} = \frac{2}{6}$$

Now add:

$$\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$$

Subtraction of Rational Numbers

Example 1

$$\frac{5}{8}-\frac{3}{8}=\frac{2}{8}=\frac{1}{4}$$

Example 2

$$\frac{3}{4}-\frac{1}{6}$$

LCM of 4 and 6 is 12.

$$\frac{3}{4}=\frac{9}{12}$$

$$\frac{1}{6}=\frac{2}{12}$$

Subtract:

$$\frac{9}{12}-\frac{2}{12}=\frac{7}{12}$$

Multiplication of Rational Numbers

Multiply numerators together and denominators together.

Example

$$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$$

Example with Negative Numbers

$$\frac{-4}{5} \times \frac{3}{8} = \frac{-12}{40} = \frac{-3}{10}$$

Division of Rational Numbers

To divide rational numbers:

1. Keep the first fraction same.
2. Change division into multiplication.
3. Take reciprocal of second fraction.

Example

$$\frac{2}{3} \div \frac{5}{7}$$

$$= \frac{2}{3} \times \frac{7}{5}$$

$$= \frac{14}{15}$$

Example 2

$$\frac{-3}{4} \div \frac{2}{5}$$

$$= \frac{-3}{4} \times \frac{5}{2}$$

$$= \frac{-15}{8}$$

Properties of Rational Numbers

(a) Closure Property:

The sum, difference, product, and quotient (except division by zero) of rational numbers is always rational.

Example

$$\frac{2}{3} + \frac{1}{5} = \frac{13}{15}$$

(b) Commutative Property:

Addition

$$a + b = b + a$$

Multiplication

$$a \times b = b \times a$$

Example

$$\frac{1}{2}+\frac{3}{4}=\frac{3}{4}+\frac{1}{2}$$

(c) Associative Property:

Addition

$$(a+b)+c=a+(b+c)$$

Multiplication

$$(a\times b)\times c=a\times(b\times c)$$

Example

$$\left(\frac{1}{2}+\frac{1}{3}\right)+\frac{1}{6}=\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{6}\right)$$

(d) Distributive Property:

$$a(b+c)=ab+ac$$

Example

$$\frac{1}{2}\left(\frac{1}{3}+\frac{2}{3}\right)$$

$$=\frac{1}{2}\times 1=\frac{1}{2}$$

(e) Identity Property:

Additive Identity

0 is additive identity.

$$a + 0 = a$$

Multiplicative Identity

1 is multiplicative identity.

$$a \times 1 = a$$

(f) Inverse Property:

Additive Inverse

Additive inverse of a is $-a$.

Multiplicative Inverse

Multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$.

Representation of Irrational Numbers on Number Line

Steps

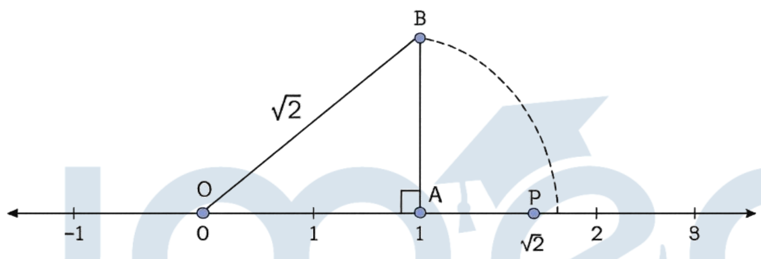
1. Draw a number line.
2. Mark point O at 0 and point A at 1.
3. At point A, draw a perpendicular AB of length 1 unit.
4. Join O and B.
5. By Pythagoras theorem:

$$OB^2 = 1^2 + 1^2$$

$$OB^2 = 2$$

$$OB = \sqrt{2}$$

6. With O as center and radius OB, cut the number line.
7. The point obtained represents $\sqrt{2}$.



Steps

1. Draw OA = 2 units on number line.
2. Draw perpendicular AB = 1 unit.
3. Join OB.
4. By Pythagoras theorem:

$$OB^2 = 2^2 + 1^2$$

$$OB^2 = 4 + 1 = 5$$

$$OB = \sqrt{5}$$

5. With O as center and OB radius, mark point on number line.
6. This point represents $\sqrt{5}$.

Finding Rational Numbers Between Given Rational Numbers

Method 1: Using Equivalent Fractions

Example

Find three rational numbers between $\frac{1}{4}$ and $\frac{3}{4}$.

Convert into equivalent fractions:

$$\frac{1}{4} = \frac{2}{8}$$

$$\frac{3}{4} = \frac{6}{8}$$

Numbers between them are:

$$\frac{3}{8}, \frac{4}{8}, \frac{5}{8}$$

Method 2: Average Method

Example

Find one rational number between $\frac{2}{5}$ and $\frac{4}{5}$.

$$\frac{\frac{2}{5} + \frac{4}{5}}{2}$$

$$= \frac{6/5}{2}$$

$$= \frac{3}{5}$$

Decimal Representation of Rational Numbers

A rational number may have:

1. Terminating decimal
2. Non-terminating recurring decimal

Terminating Decimal:

A decimal that ends after finite digits.

Examples

$$\frac{1}{2} = 0.5$$

$$\frac{3}{4} = 0.75$$

Recurring Decimal:

A decimal in which one or more digits repeat forever.

Examples

$$\frac{1}{3} = 0.3333\ldots$$

$$\frac{2}{11} = 0.181818\ldots$$

Converting Fractions into Decimals

Use long division:

Example 1

Convert $\frac{3}{8}$ into decimal.

$$3 \div 8 = 0.375$$

Example 2

Convert $\frac{5}{6}$ into decimal.

$$5 \div 6 = 0.8333\ldots$$

Converting Recurring Decimals into Fractions

Example 1

Convert 0.3333...into fraction.

Let:

$$x = 0.3333\ldots$$

Multiply by 10:

$$10x = 3.3333...$$

Subtract:

$$10x - x = 3.3333... - 0.3333...$$

$$9x = 3$$

$$x = \frac{3}{9} = \frac{1}{3}$$

Example 2

Convert 0.727272. ..into fraction.

Let:

$$x = 0.727272...$$

Multiply by 100:

$$100x = 72.727272...$$

Subtract:

$$100x - x = 72$$

$$99x = 72$$

$$x = \frac{72}{99} = \frac{8}{11}$$

Detailed Explanation of Irrational Numbers

An irrational number is a number that cannot be expressed in the form:

$$\frac{p}{q}$$

where p and q are integers and q $\neq$ 0.

Important Features

- Decimal expansion is non-terminating.
- Decimal expansion is non-recurring.
- Cannot be written as exact fractions.

Examples

$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$$

Why Are They Important?

Irrational numbers help in:

- Geometry
- Measurement
- Trigonometry
- Engineering
- Physics

For example:

- Diagonal of a square involves $\sqrt{2}$
- Circumference of circle involves π

Proof of Irrationality

Assume $\sqrt{2}$ is rational.

Then:

$$\sqrt{2} = \frac{p}{q}$$

where p and q are integers having no common factor.

Squaring both sides:

$$2 = \frac{p^2}{q^2}$$

$$p^2 = 2q^2$$

So, p^2 is even.

Therefore, p is even.

Let:

$$p = 2k$$

Substitute:

$$(2k)^2 = 2q^2$$

$$4k^2 = 2q^2$$

$$2k^2 = q^2$$

Thus, q^2 is even.

Therefore, q is also even.

This means both p and q are even.

So, p and q have common factor 2.

But this contradicts our assumption that p and q have no common factor.

Hence, $\sqrt{2}$is irrational.

Introduction to Decimal Expansion

Decimal expansion means expressing a number in decimal form.

A decimal number is obtained when a number is written using digits and a decimal point.

Examples:

- $5 = 5.0$
- $\frac{1}{2} = 0.5$
- $\frac{3}{4} = 0.75$

Decimal expansion helps us:

- compare numbers easily,
- perform calculations,
- represent fractions in simpler forms.

Decimal Number

A decimal number has two parts:

1. Whole number part
2. Decimal part

Example:

$$45.678$$

- 45 → whole number part
- 678 → decimal part

The decimal point separates the two parts.

Place Value in Decimal Numbers:

Place	Value
Ones	1
Tenths	$\frac{1}{10}$
Hundredths	$\frac{1}{100}$
Thousandths	$\frac{1}{1000}$

Example: 3.472

- 4 is in tenths place
- 7 is in hundredths place
- 2 is in thousandths place

Expanded form:

$$3 + \frac{4}{10} + \frac{7}{100} + \frac{2}{1000}$$

Decimal Expansion of Rational Numbers

A rational number can always be written in decimal form.

A rational number may have:

1. Terminating decimal expansion
2. Non-terminating recurring decimal expansion

Terminating Decimal Expansion:

A decimal that ends after a finite number of digits is called a terminating decimal.

Examples

$$\frac{1}{2} = 0.5$$
$$\frac{3}{4} = 0.75$$
$$\frac{7}{8} = 0.875$$

These decimals stop after some digits.

Non-Terminating Recurring Decimal Expansion:

A decimal that continues forever with repeating digits is called a recurring decimal.

Examples

$$\frac{1}{3} = 0.3333\ldots$$
$$\frac{2}{11} = 0.181818\ldots$$

$$\frac{5}{6} = 0.83333\ldots$$

The repeating digit or group of digits is called the repetend.

How to Identify Terminating and Recurring Decimals

For a rational number:

$$\frac{p}{q}$$

First simplify the fraction completely.

Rule

If denominator has only factors:

$$2 \text{ and/or } 5$$

then decimal expansion is terminating.

Otherwise, decimal expansion is recurring.

Examples of Terminating Decimals

Example 1

$$\frac{3}{8}$$

Factorize denominator:

$$8 = 2^3$$

Only factor is 2.

Therefore decimal expansion terminates.

$$\frac{3}{8} = 0.375$$

Example 2

$$\frac{7}{20}$$
$$20 = 2^2 \times 5$$

Only factors are 2 and 5.

Hence terminating decimal.

$$\frac{7}{20} = 0.35$$

Examples of Recurring Decimals:

Example 1

$$\frac{1}{3}$$

Denominator:

$$3$$

Factor is neither 2 nor 5.

So decimal expansion is recurring.

$$0.3333\ldots$$

Example 2

$$\frac{2}{7}$$

Denominator:

$$7$$

Since 7 is not 2 or 5, decimal expansion is recurring.

$$0.285714285714\ldots$$

Converting Fractions into Decimal Expansion

Use long division.

Example 1

Convert: $\frac{1}{4}$

Step 1

Divide 1 by 4.

Step 2

Add decimal point and zeros.

$$1.000 \div 4$$

Result

$$0.25$$

Example 2

Convert:

$$\frac{5}{8}$$

Long division gives:

$$0.625$$

Example 3

Convert:

$$\frac{2}{3}$$

Long division gives:

$$0.6666\ldots$$

This is recurring decimal expansion.

Pure Recurring Decimals:

If repetition starts immediately after decimal point, it is called pure recurring decimal.

Examples

$$0.3333\ldots$$
$$0.727272\ldots$$
$$0.9999\ldots$$

Mixed Recurring Decimals:

If some digits are non-repeating before repeating begins, it is called mixed recurring decimal.

Examples

$$0.16666\ldots$$
$$0.245555\ldots$$
$$0.1232323\ldots$$

Converting Recurring Decimals into Fractions:

Example 1

Convert:

$$0.3333\ldots$$

Let:

$$x = 0.3333\ldots$$

Multiply by 10:

$$10x = 3.3333\ldots$$

Subtract:

$$10x - x = 3$$
$$9x = 3$$
$$x = \frac{1}{3}$$

Example 2

Convert:

$$0.727272\ldots$$

Let:

$$x = 0.727272\ldots$$

Multiply by 100:

$$100x = 72.727272\ldots$$

Subtract:

$$100x - x = 72$$
$$99x = 72$$
$$x = \frac{72}{99}$$
$$x = \frac{8}{11}$$

Decimal Expansion of Irrational Numbers

Irrational numbers have:

- non-terminating

- non-recurring

decimal expansions.

Examples

$$\sqrt{2} = 1.41421356\ldots$$
$$\sqrt{5} = 2.2360679\ldots$$
$$\pi = 3.14159265\ldots$$

Digits never stop and never repeat in a fixed pattern.

Difference Between Rational and Irrational Decimal Expansion:

Rational Numbers	Irrational Numbers
Decimal expansion terminates or repeats	Decimal expansion neither terminates nor repeats
Can be written as fractions	Cannot be written as fractions
Examples: 0.5, 0.333...	Examples: $\sqrt{2}, \pi$

Important Results About Decimal Expansion

Result 1

Every rational number has:

- terminating OR
- recurring decimal expansion.

Result 2

Every irrational number has:

- non-terminating,
- non-recurring decimal expansion.

Real-Life Applications of Decimal Expansion:

Decimal numbers are used in:

- money calculations,
- measurements,
- science,
- engineering,
- banking,
- statistics,
- shopping.

Examples:

- ₹25.75
- 1.75 m
- 98.6°F

Solved Examples

Example 1

State whether decimal expansion terminates:

$$\frac{9}{40}$$

Factorize denominator:

$$40 = 2^3 \times 5$$

Only factors 2 and 5.

Hence terminating decimal.

Example 2

State whether decimal expansion recurs:

$$\frac{7}{12}$$
$$12 = 2^2 \times 3$$

Factor 3 present.

Hence recurring decimal.

Example 3

Convert:

$$\frac{11}{20}$$
$$11 \div 20 = 0.55$$

Example 4

Convert recurring decimal into fraction:

$$0.4444\ldots$$

Let:

$$x = 0.4444\ldots$$
$$10x = 4.4444\ldots$$

Subtract:

$$9x = 4$$
$$x = \frac{4}{9}$$

Practice Worksheet

Section A: MCQs

1. Which of the following is a rational number?

a) $\sqrt{2}$

b) π

c) $\frac{3}{5}$

d) $\sqrt{7}$

2. Which number is irrational?

a) $\frac{7}{8}$

b) 0.25

c) $\sqrt{3}$

d) -5

3. Equivalent fraction of $\frac{2}{3}$ is:

a) $\frac{4}{5}$

b) $\frac{6}{9}$

c) $\frac{3}{5}$

d) $\frac{5}{7}$

4. $\frac{1}{2}+\frac{1}{3}=$

a) $\frac{2}{5}$

b) $\frac{5}{6}$

c) $\frac{1}{6}$

d) $\frac{3}{5}$

5. $\frac{5}{6}-\frac{1}{3}=$

a) $\frac{1}{2}$

b) $\frac{2}{3}$

c) $\frac{1}{3}$

d) $\frac{5}{9}$

6. $\frac{2}{5}\times\frac{3}{4}=$

a) $\frac{5}{10}$

b) $\frac{6}{4}$

c) $\frac{-6}{10}$

d) $\frac{5}{9}$

7. Reciprocal of $\frac{7}{9}$ is:

a) $\frac{9}{7}$

b) $\frac{7}{9}$

c) $\frac{-9}{7}$

d) $\frac{1}{7}$

8. Decimal form of $\frac{1}{4}$ is:

a) 0.4

b) 0.25

c) 2.5

d) 0.75

9. Which property is shown by:

$$a + b = b + a$$

a) Associative Property of Addition

b) Commutative Property of Addition

c) Distributive Property

d) Identity Property of Addition

10. Which of the following is terminating?

a) 0.333...

b) 0.272727...

c) 0.875

d) 0.181818...

11. $\sqrt{5}$ is:

a) Natural

b) Rational

c) Irrational

d) Integer

12. Additive identity is:

a) 0

b) 1

c) -1

d) 10

13. Multiplicative identity is:

a) 0

b) 1

c) 2

d) -1

14. $\frac{2}{7} \div \frac{3}{5} =$

a) $\frac{6}{35}$

b) $\frac{10}{21}$

c) $\frac{21}{10}$

d) $\frac{35}{6}$

15. Which set contains only integers?

a) 1, 2, 3

b) -2, 0, 5

c) $\frac{1}{2}, 3$

d) $\sqrt{2}, 4$

16. The decimal expansion of irrational numbers is:

a) terminating

b) recurring

c) non-terminating non-recurring

d) finite

17. Which is not rational?

a) -7

b) 0

c) 0.5

d) $\sqrt{11}$

18. $\frac{3}{8} =$

a) 0.375

b) 3.75

c) 0.38

d) 0.83

19. The reciprocal of -5 is:

a) 5

b) -5

c) $\frac{-1}{5}$

d) $\frac{1}{5}$

20. Rational numbers can be represented in the form: $\frac{p}{q}$ where:

a) $q = 0$

b) $q \neq 0$

c) $p = 0$ only

d) p and q are natural numbers only

Section B: Fill in the Blanks

1. Every integer is a ________ number.
2. The reciprocal of $\frac{5}{9}$is ________.
3. $\sqrt{2}$is an ________ number.
4. Additive identity of rational numbers is ________.
5. Multiplicative identity is ________.
6. Decimal expansion of irrational numbers is non-terminating and ________.
7. $\frac{1}{2}$ =________ in decimal form.
8. $\frac{2}{3}+\frac{1}{3}$ =________.
9. $\frac{5}{6}-\frac{1}{6}$ =________.
10. Reciprocal of $\frac{-3}{7}$is ________.
11. 0.333… =________ as fraction.
12. Rational numbers are closed under ________.
13. The denominator of a rational number cannot be ________.
14. $\sqrt{5}$lies between ________ and ________.
15. 0 is a ________ number.
16. Whole numbers include ________.
17. $\frac{3}{4}\times\frac{2}{5}$ =________.
18. $\frac{7}{8}\div\frac{1}{2}$ =________.
19. $\pi$is an ________ number.

20. Decimal form of $\frac{1}{8}$ is ________.

Section C: True/False (10 Questions)

1. Every rational number is an integer.
2. $\sqrt{2}$is irrational.
3. Rational numbers can have recurring decimals.
4. Zero is a whole number.
5. Irrational numbers can be written as fractions.
6. $\frac{1}{2}+\frac{1}{2}=1$
7. Multiplication of rational numbers is commutative.
8. 0.125is a terminating decimal.
9. $\sqrt{9}=3$
10. The reciprocal of 0 exists.

Section D: Short Answer Questions

1. Define rational numbers with examples.
2. Define irrational numbers.
3. Find three equivalent rational numbers of $\frac{4}{7}$.
4. Add:

$$\frac{2}{5}+\frac{3}{10}$$

5. Subtract:

$$\frac{7}{8} - \frac{1}{4}$$

6. Multiply:

$$\frac{3}{7} \times \frac{14}{9}$$

7. Divide:

$$\frac{5}{6} \div \frac{2}{3}$$

8. Convert $\frac{7}{20}$ into decimal.
9. Convert 0.666... into fraction.
10. Prove that $\sqrt{2}$is irrational.
11. Find two rational numbers between $\frac{1}{3}$and $\frac{2}{3}$.
12. State any four properties of rational numbers.

Answer Key

MCQ Answers

1. c
2. c
3. b
4. b
5. a
6. $\frac{3}{10}$
7. a
8. b
9. Commutative property
10. c
11. c
12. a
13. b
14. $\frac{10}{21}$
15. b
16. c
17. d
18. a
19. c
20. b

Fill in the Blanks Answers

1. rational
2. $\frac{9}{5}$
3. irrational
4. 0
5. 1
6. non-recurring
7. 0.5
8. 1
9. $\frac{2}{3}$
10. $\frac{-7}{3}$
11. $\frac{1}{3}$
12. operations
13. zero
14. 2 and 3
15. whole
16. zero

17. $\frac{3}{10}$

18. $\frac{7}{4}$

19. irrational

20. 0.125

True/False Answers

1. False
2. True
3. True
4. True
5. False
6. True
7. True
8. True
9. True
10. False
11.

Short Answer Hints

1. Rational numbers are numbers of the form $\frac{p}{q}$, where q ≠ 0.
2. Irrational numbers cannot be written in fractional form.
3. $\frac{8}{14}, \frac{12}{21}, \frac{16}{28}$
4. $\frac{7}{10}$
5. $\frac{5}{8}$
6. $\frac{2}{3}$
7. $\frac{5}{4}$
8. 0.35
9. $\frac{2}{3}$

10. Use contradiction method.

11. $\frac{4}{9}, \frac{5}{9}$

12. Closure, commutative, associative, distributive.

End of Chapter