

Comprehensive Notes
of
Central Board of Secondary Education

Class IX

Mathematics
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Chapter -4 (Exploring Algebraic Identities)

by

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Importance of Algebraic Identities:

We use algebraic identities because they help us solve mathematical problems quickly, accurately, and easily without doing long calculations every time.

Further they are useful to:

- Save time
- Reduce calculation work
- Make factorization easier
- Help solve equations quickly
- Improve mental maths
- Build the foundation for higher mathematics

Some important algebraic identities:

- I. $(a + b)^2 = a^2 + 2ab + b^2$
- II. $(a - b)^2 = a^2 - 2ab + b^2$
- III. $a^2 - b^2 = (a - b)(a + b)$
- IV. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- V. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- VI. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- VII. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Identity 1:

Square of Sum

$$(a + b)^2 = a^2 + 2ab + b^2$$

Explanation: When a binomial is multiplied by itself:

$$(a + b)(a + b)$$

we get:

- Square of first term
- Plus, twice the product
- Plus, square of second term

Example 1:

Expand: $(x + 5)^2$

Using identity:

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 \\ &= x^2 + 2(x)(5) + 5^2 \\ &= x^2 + 10x + 25\end{aligned}$$

Example 2:

Find: $(2a + 3b)^2$

$$\begin{aligned}&= (2a)^2 + 2(2a)(3b) + (3b)^2 \\ &= 4a^2 + 12ab + 9b^2\end{aligned}$$

Identity 2:

Square of Difference

$$(a - b)^2 = a^2 - 2ab + b^2$$

Example:Expand: $(3x - 2)^2$

$$\begin{aligned} &= (3x)^2 - 2(3x)(2) + 2^2 \\ &= 9x^2 - 12x + 4 \end{aligned}$$

Identity 3:

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$

Example:Factorize: $x^2 - 49$

Since:

$$x^2 - 7^2 = (x - 7)(x + 7)$$

Identity 4:

Cube of Sum

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Example:

Expand: $(x + 2)^3$

$$\begin{aligned} &= x^3 + 3x^2(2) + 3x(2^2) + 2^3 \\ &= x^3 + 6x^2 + 12x + 8 \end{aligned}$$

Identity 5 :

Cube of Difference

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Example:

Expand: $(2x - 1)^3$

$$= 8x^3 - 12x^2 + 6x - 1$$

Identity 6:

Sum of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Example:

Factorize: $x^3 + 27$

Since:

$$\begin{aligned} a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\ &= x^3 + 3^3 \\ &= (x + 3)(x^2 - 3x + 9) \end{aligned}$$

Identity 7 :

Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example:Factorize: $8x^3 - 1$

$$\begin{aligned} &= (2x)^3 - 1^3 \\ &= (2x - 1)(4x^2 + 2x + 1) \end{aligned}$$

Finding Squares of Numbers Using Identities**Solve $(64)^2$**

Use:

$$(60 + 4)^2$$

Using identity:

$$\begin{aligned} &a^2 + 2ab + b^2 \\ &= 60^2 + 2(60)(4) + 4^2 \\ &= 3600 + 480 + 16 \\ &= 4096 \end{aligned}$$

Therefore,

$$(64)^2 = 4096$$

Solve $(98)^2$

$$(100 - 2)^2$$

$$\begin{aligned} &= 10000 - 400 + 4 \\ &= 9604 \end{aligned}$$

Solve $(105)^2$

$$\begin{aligned} &(100 + 5)^2 \\ &= 10000 + 1000 + 25 \\ &= 11025 \end{aligned}$$

Identity from Given Equation

Example 1:

Identify the identity:

$$x^2 + 2xy + y^2$$

Compare with:

$$a^2 + 2ab + b^2$$

Hence,

$$= (x + y)^2$$

Example 2:

Identify:

$$9a^2 - 25$$

$$= (3a)^2 - 5^2$$

Using difference of squares:

$$= (3a - 5)(3a + 5)$$

Factorization Using Identities

Example 1:

Factorize:

$$x^2 + 10x + 25$$

Compare with:

$$a^2 + 2ab + b^2$$

Here:

- $a = x$
- $b = 5$

Therefore,

$$= (x + 5)^2$$

Example 2:

Factorize: $4x^2 - 12x + 9$

Comparing with

$$\begin{aligned}(a - b)^2 &= a^2 - 2ab + b^2 \\ &= (2x)^2 - 2(2x)(3) + 3^2 \\ &= (2x - 3)^2\end{aligned}$$

Example 3:

Factorize: $16a^2 - 81$

$$\begin{aligned}&= (4a)^2 - 9^2 \\ &= (4a - 9)(4a + 9)\end{aligned}$$

Factorization Without Algebra Tiles

Factorization can also be done by:

- Splitting middle term
- Common factors
- Using identities directly

Example:

Factorize:

$$x^2 + 7x + 12$$

Find two numbers whose:

- Product = 12
- Sum = 7

Numbers are 3 and 4.

$$=x^2 + 3x + 4x + 12$$

$$= x(x + 3) + 4(x + 3)$$

$$= (x + 3)(x + 4)$$

Simplifying Rational Expressions by Factors

A rational expression contains fractions with variables.

Example 1

Simplify:

$$\frac{x^2 - 25}{x - 5}$$

Factor numerator:

$$\frac{x^2 - 25 = (x - 5)(x + 5)}{(x - 5)(x + 5)} \\ \frac{(x - 5)(x + 5)}{x - 5}$$

Cancel common factor:

$$= x + 5$$

Example 2

Simplify:

$$\frac{x^2 + 6x + 9}{x + 3}$$

Factor numerator:

$$= (x + 3)^2$$

$$= \frac{(x + 3)(x + 3)}{x + 3}$$
$$= x + 3$$

Solved Questions

Q1. Expand

$$(5x + 2)^2$$

Solution

$$= 25x^2 + 20x + 4$$

Q2. Factorize

$$y^2 - 16$$

Solution

$$= (y - 4)(y + 4)$$

Q3. Find the square of 103

Solution

$$\begin{aligned}(100 + 3)^2 \\ &= 10000 + 600 + 9 \\ &= 10609\end{aligned}$$

Q4. Simplify

$$\frac{x^2 - 9}{x - 3}$$

Solution

$$\begin{aligned}&= \frac{(x - 3)(x + 3)}{x - 3} \\ &= x + 3\end{aligned}$$

Practice Worksheet

Section A – MCQ

1. $(a+b)^2 =$

a) $a^2 + b^2$

b) $a^2 + 2ab + b^2$

c) $a^2 - 2ab + b^2$

d) $2ab$

2. $x^2 - 25 =$

a) $(x - 5)^2$

b) $(x + 5)^2$

c) $(x - 5)(x + 5)$

d) None

3. $64^2 =$

a) 4096

b) 4069

c) 4906

d) 6400

4. $(a-b)^2 =$

a) $a^2 + b^2$

b) $a^2 - 2ab + b^2$

c) $a^2 + 2ab + b^2$

d) $2ab$

5. $x^2 + 10x + 25 =$

a) $(x + 5)^2$

b) $(x - 5)^2$

c) $(x + 25)^2$

d) None

6. $a^2 - b^2 =$

a) $(a - b)^2$

b) $(a + b)^2$

c) $(a - b)(a + b)$

d) None

7. $(2x+3)^2 =$

a) $4x^2 + 12x + 9$

b) $4x^2 + 6x + 9$

c) $2x^2 + 9$

d) None

8. $81 - 16 =$

a) $(9 - 4)^2$

b) $(9 + 4)^2$

c) $(9 - 4)(9 + 4)$

d) None

9. $(x+2)^3 =$

a) $x^3 + 6x^2 + 12x + 8$

b) $x^3 + 8$

c) $x^3 + 4$

d) None

10. $125 =$

a) 5^2

b) 5^3

c) 25^2

d) 15^2

Section B – Fill in the Blanks

1. $(a+b)^2 =$ _____

2. $(a-b)^2 =$ _____

3. $a^2 - b^2 =$ _____

4. $(x+4)^2 =$ _____

5. $(x-5)^2 = \underline{\hspace{2cm}}$

6. $81 - 49 = \underline{\hspace{2cm}}$

7. $(2a+3)^2 = \underline{\hspace{2cm}}$

8. $(x+y)^3 = \underline{\hspace{2cm}}$

9. $64^2 = \underline{\hspace{2cm}}$

10. $99^2 = \underline{\hspace{2cm}}$

11. $x^2 + 8x + 16 = \underline{\hspace{2cm}}$

12. $25a^2 - 9 = \underline{\hspace{2cm}}$

13. $(3x-1)^2 = \underline{\hspace{2cm}}$

14. $(a + b)(a - b) = \underline{\hspace{2cm}}$

15. $x^3 + 8 = \underline{\hspace{2cm}}$

16. $x^3 - 27 = \underline{\hspace{2cm}}$

17. $\frac{x^2-9}{x-3} = \underline{\hspace{2cm}}$

18. $\frac{x^2+6x+9}{x+3} = \underline{\hspace{2cm}}$

19. $144^2 = \underline{\hspace{2cm}}$

20. $125x^3 - y^3 = \underline{\hspace{2cm}}$

Section C – True / False

1. $(a + b)^2 = a^2 + b^2$

2. $a^2 - b^2 = (a - b)(a + b)$

3. $64^2 = 4096$

4. $x^2 + 4x + 4 = (x + 2)^2$

5. $x^2 - 16 = (x - 4)^2$

6. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

7. $(2x - 3)^2 = 4x^2 - 12x + 9$

8. $81 - 25 = (9 - 5)(9 + 5)$

9. $\frac{x^2-25}{x-5} = x + 5$

10. $100^2 = 1000$

Section D – Expansion of Identities

Expand the following:

1. $(x + 3)^2$

2. $(2a - 5)^2$

3. $(x + 2)^3$

4. $(3x + 4)^2$

5. $(a - b)^2$

6. $(2x - 1)^3$

7. $(5a + 2b)^2$

8. $(x - 7)^2$

9. $(3p + q)^3$

10. $(4x - 3y)^2$

Answer Key

MCQ Answers

1- b

2- c

3- a

4- b

5- a

6- c

7- a

8- c

9- a

10- b

Section B : Fill in the Blanks Answers

1. $a^2 + 2ab + b^2$

2. $a^2 - 2ab + b^2$

3. $(a - b)(a + b)$

4. $x^2 + 8x + 16$

5. $x^2 - 10x + 25$

6. $(9 - 7)(9 + 7)$

7. $4a^2 + 12a + 9$

8. $x^3 + 3x^2y + 3xy^2 + y^3$

9. 4096

10. 9801

11. $(x + 4)^2$

12. $(5a - 3)(5a + 3)$

13. $9x^2 - 6x + 1$

14. $a^2 - b^2$

15. $(x + 2)(x^2 - 2x + 4)$
16. $(x - 3)(x^2 + 3x + 9)$
17. $x + 3$
18. $x + 3$
19. 20736
20. $(5x - y)(25x^2 + 5xy + y^2)$

Section C: True / False Answers

- | | | |
|----------|----------|-----------|
| 1. False | 5. False | 9. True |
| 2. True | 6. True | 10. False |
| 3. True | 7. True | |
| 4. True | 8. True | |

Section D – Expansion of Identities

- | | |
|---------------------------|-----------------------------------|
| 1. $x^2 + 6x + 9$ | 6. $8x^3 - 12x^2 + 6x - 1$ |
| 2. $4a^2 - 20a + 25$ | 7. $25a^2 + 20ab + 4b^2$ |
| 3. $x^3 + 6x^2 + 12x + 8$ | 8. $x^2 - 14x + 49$ |
| 4. $9x^2 + 24x + 16$ | 9. $27p^3 + 27p^2q + 9pq^2 + q^3$ |
| 5. $a^2 - 2ab + b^2$ | 10. $16x^2 - 24xy + 9y^2$ |

End of Chapter