Comprehensive Notes

of

Central Board of Secondary Education

Class IX

Mathematics

(Ganita Manjari)

Chapter - 8 (Exploring Sequences)

by

lumens

Introduction to Sequences

A sequence is an ordered list of numbers arranged according to a rule.

Each number in the sequence is called a term.

Example:

$$2,\ 4,\ 6,\ 8,\ 10,\ \ldots$$

Here:

- 2 is the first term
- 4 is the second term
- 6 is the third term

The position of a term is important in a sequence.

Real-Life Examples of Sequences

Example 1: Counting Numbers

$$1,\ 2,\ 3,\ 4,\ 5,\ \ldots$$

Example 2: Even Numbers

$$2,\ 4,\ 6,\ 8,\ 10,\ \ldots$$

Example 3: Multiples of 5

$$5,\ 10,\ 15,\ 20,\ \ldots$$

Example 4: Pattern in Nature

Petals in flowers often follow special sequences like the Fibonacci sequence.

Terms of a Sequence

The terms are represented using symbols:

$$a_1,\ a_2,\ a_3,\ a_4, \ldots$$

Where:

- a_1= first term
- a_2= second term
- a_n= nth term

Example

Sequence:

$$3,\ 6,\ 9,\ 12,\ 15, \ldots$$

Then:

- $a_1 = 3$
- $a_2 = 6$
- $a_3 = 9$
- $a_5 = 15$

Explicit Rule for a Sequence

An explicit rule gives the nth term directly without finding earlier terms.

It is also called the general formula.

Example 1

Sequence:

$$2,\ 4,\ 6,\ 8,\ 10, \ldots$$

Observe:

- First term $= 2$
- Second term $= 2 \times 2$
- Third term $= 2 \times 3$

Thus,

$$a_n = 2n$$

Finding Terms Using Explicit Rule:

If:

$$a_n = 3n + 1$$

Find first five terms.

Solution

For $n = 1$:

$$a_1 = 3(1) + 1 = 4$$

For $n = 2$:

$$a_2 = 3(2) + 1 = 7$$

For $n = 3$:

$$a_3 = 10$$

For $n = 4$:

$$a_4 = 13$$

For $n = 5$:

$$a_5 = 16$$

Sequence:

$$4,\ 7,\ 10,\ 13,\ 16$$

Recursive Rule for a Sequence

A recursive rule defines a term using previous terms.

It tells:

1. The first term
2. How to get the next term

Example 1

Sequence:

$$5,\ 8,\ 11,\ 14, \ldots$$

Recursive rule:

- First term:

$$a_1 = 5$$

- Add 3 each time:

$$a_n = a_{n-1} + 3$$

Example 2

Sequence:

$$2,\ 4,\ 8,\ 16, \ldots$$

Recursive formula:

$$a_1 = 2$$
$$a_n = 2a_{n-1}$$

Each term is double the previous term.

Virahānka–Fibonacci Sequence

The Virahānka–Fibonacci sequence was studied in India long before Fibonacci.

The sequence is:

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21, \ldots$$

Each term is obtained by adding the previous two terms.

Recursive Formula:

$$F_1 = 1, F_2 = 1$$
$$F_n = F_{n-1} + F_{n-2}$$

Example

$$1,\ 1,\ 2,\ 3,\ 5, \ldots$$

Because:

$$1 + 1 = 2$$
$$1 + 2 = 3$$
$$2 + 3 = 5$$

Applications of Fibonacci Sequence:

- Nature
- Pine cones
- Flower petals
- Spiral shells
- Computer algorithms

Arithmetic Progressions (AP)

An Arithmetic Progression is a sequence where the difference between consecutive terms is constant.

This constant difference is called the common difference.

General Form

$$a,\ a+d,\ a+2d,\ a+3d,\ldots$$

Where:

- a= first term
- d= common difference

Example 1

$$2,\ 5,\ 8,\ 11,\ 14$$

Differences:

$$5-2=3$$
$$8-5=3$$

So,

$$d=3$$

Hence, it is an AP.

Formula for nth Term of AP

$$a_n = a + (n-1)d$$

Example

Find the 10th term of:

$$3,\ 7,\ 11,\ 15, \ldots$$

Solution

Here:

$$a = 3, d = 4, n = 10$$

Using formula:

$$a_n = a + (n-1)d$$
$$a_{10} = 3 + (10-1) \times 4$$
$$= 3 + 36$$
$$= 39$$

Visualising an AP

Consider:

$$1,\ 3,\ 5,\ 7, \ldots$$

These numbers can be shown using dots.

1st term

•
(1 dot)

2nd term

•••
(3 dots)

3rd term

•••••
(5 dots)

Each row increases by 2 dots.

Thus the common difference is 2.

Sum of the First n Natural Numbers

Natural numbers:

$$1,\ 2,\ 3,\ 4, \ldots$$

The sum is:

$$1 + 2 + 3 + \cdots + n$$

Formula:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Example

Find:

$$1 + 2 + 3 + \cdots + 20$$

Solution

Using formula:

$$\frac{20(21)}{2}$$
$$= 10 \times 21$$
$$= 210$$

Triangular Number Sequence

The triangular numbers are formed by arranging dots in triangles.

Sequence:

$$1,\ 3,\ 6,\ 10,\ 15, \ldots$$

Formation

$$1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$1 + 2 + 3 + 4 = 10$$

Formula

$$T_n = \frac{n(n+1)}{2}$$

Example:

Find the 7th triangular number.

Solution

$$T_7 = \frac{7(8)}{2}$$
$$= 28$$

Geometric Progressions (GP)

A Geometric Progression is a sequence where each term is multiplied by a fixed number.

This fixed number is called the common ratio.

General Form

$$a,\ ar,\ ar^2,\ ar^3, \ldots$$

Where:

- a= first term
- r= common ratio

Common Ratio:

The ratio between consecutive terms is constant.

$$r = \frac{\text{next term}}{\text{previous term}}$$

Example

$$3,\ 6,\ 12,\ 24, \ldots$$

$$r = \frac{6}{3} = 2$$

$$r = \frac{12}{6} = 2$$

Hence common ratio:

$$r = 2$$

nth Term of GP

$$a_n = ar^{n-1}$$

Example

Find the 6th term of:

$$2,\ 4,\ 8,\ 16, \ldots$$

Solution

$$a = 2, r = 2, n = 6$$
$$a_n = ar^{n-1}$$
$$a_6 = 2(2)^5$$
$$= 2 \times 32$$
$$= 64$$

Visualising a GP:

Sequence:

$$1,\ 2,\ 4,\ 8,\ 16$$

Using squares:

- 1 square
- 2 squares
- 4 squares
- 8 squares

Each stage doubles.

Thus:

$$r = 2$$

Difference Between AP and GP

AP	GP
Constant difference	Constant ratio
Add same number	Multiply by same number
Example: 2,4,6,8	Example: 2,4,8,16

Recursive Formula for AP and GP:

Recursive Formula of AP

$$a_1 = a$$
$$a_n = a_{n-1} + d$$

Recursive Formula of GP

$$a_1 = a$$
$$a_n = ra_{n-1}$$

Solved Examples

Example 1

Find the next term:

$$4,\ 7,\ 10,\ 13, \ldots$$

Solution

Common difference:

$$7 - 4 = 3$$

Next term:

$$13 + 3 = 16$$

Answer:

$$16$$

Example 2

Find the common ratio:

$$5,\ 15,\ 45, \ldots$$

Solution

$$r = \frac{15}{5} = 3$$

Answer:

3

Example 3

Find the 15th term of AP:

$$2,\ 5,\ 8, \ldots$$

Solution

$$a = 2, d = 3, n = 15$$
$$a_n = a + (n-1)d$$
$$= 2 + (14)(3)$$
$$= 44$$

Practice Test Paper

Section A – MCQs

1. Which of the following is a sequence?
   a) 2,4,6,8
   b) A,B,C
   c) 1,3,5,7
   d) All of these

2. The common difference of: 5,8,11,14 is:
   a) 2
   b) 3
   c) 4
   d) 5

3. Which is a GP?
   a) 2,4,6,8
   b) 3,6,12,24
   c) 5,10,15
   d) 1,4,9

4. Fibonacci sequence starts with:
   a) 1,2
   b) 0,1
   c) 1,1
   d) 2,3

5. nth term of AP is:
   a) $a + r(n - 1)$

b) $a + (n-1)d$

c) ar^n

d) $a + d^n$

6. Common ratio of: 2,6,18 is:

a) 2

b) 3

c) 6

d) 9

7. The 5th triangular number is:

a) 10

b) 15

c) 20

d) 25

8. Which sequence increases by multiplication?

a) AP

b) GP

c) Fibonacci

d) Natural numbers

9. The sequence: 1,4,9,16 represents:

a) cubes

b) squares

c) primes

d) odds

10. Recursive formula uses:

a) previous terms

b) geometry

c) algebra only

d) graphs

11. The first natural number is:

a) 0

b) 1

c) 2

d) 10

12. In GP: 3,9,27 common ratio is:

a) 2

b) 3

c) 6

d) 9

13. The sum: $1 + 2 + 3 + 4$ equals:

a) 8

b) 9

c) 10

d) 12

14. AP means:

a) Arithmetic Progression

b) Algebraic Pattern

c) Added Product

d) Arithmetic Product

15. GP means:

a) General Pattern

b) Geometric Progression

c) Great Product

d) Graphical Pattern

16. The sequence: 2,4,8,16 has common ratio:

a) 2

b) 4

c) 8

d) 16

17. Fibonacci sequence is formed by:

a) subtraction

b) division

c) adding previous two terms

d) multiplication

18. nth term of GP is:

a) $a + (n - 1)d$

b) ar^{n-1}

c) $n + r$

d) $a + r$

19. The sequence: 1,3,6,10 is called:

a) square numbers

b) triangular numbers

c) odd numbers

d) prime numbers

20. The sequence: 10,20,30 is:

a) GP

b) AP

c) Fibonacci

d) triangular

Section B – Fill in the Blanks

1. A sequence is an _____ list of numbers.
2. The fixed difference in AP is called _____.
3. The fixed multiplier in GP is called _____.
4. Fibonacci sequence begins with _____ and _____.
5. The formula of nth term of AP is _____.
6. The nth term of GP is _____.
7. The sequence 2,4,6,8is an _____.
8. The sequence 3,6,12,24is a _____.
9. Recursive formula depends on _____ terms.
10. The sum of first n natural numbers is _____.
11. Triangular numbers are formed using _____ arrangement.
12. The common ratio of 2,6,18is _____.
13. The common difference of 5,10,15is _____.
14. Fibonacci sequence is also called _____ sequence.
15. The first triangular number is _____.
16. In GP each term is obtained by _____.
17. AP increases by _____.
18. GP increases by _____.
19. The 4th triangular number is _____.
20. The sequence 1,2,4,8is a _____.

Section C – True / False

1. Every GP is an AP.
2. Fibonacci sequence uses addition.
3. AP has a constant difference.
4. GP has a constant ratio.
5. Recursive rules use previous terms.
6. 2,4,8,16is an AP.
7. 5,10,15,20is a GP.
8. Triangular numbers are formed using sums of natural numbers.
9. 1,1,2,3,5is Fibonacci sequence.
10. Explicit rule directly gives nth term.

Section D – Short Answer Questions

1. Find the next three terms:

   2,5,8,11, …

2. Find common ratio:

   4,12,36, …

3. Find 12th term of AP:

   3,7,11, …

4. Find 6th term of GP:

   5,10,20, …

5. Write first six Fibonacci numbers.
6. Find sum of first 15 natural numbers.
7. Find the 8th triangular number.
8. Write recursive formula for:

$$2,5,8,11, \ldots$$

9. Write explicit rule for:

$$4,8,12,16, \ldots$$

10. State difference between AP and GP.

Answer Key

MCQs

1-d	8-b	15-b
2-b	9-b	16-a
3-b	10-a	17-c
4-c	11-b	18-b
5-b	12-b	19-b
6-b	13-c	20-b
7-b	14-a	

Fill in the Blanks

1. ordered
2. common difference
3. common ratio
4. 1, 1
5. $a + (n - 1)d$
6. ar^{n-1}
7. AP
8. GP
9. previous
10. $\frac{n(n+1)}{2}$
11. triangular
12. 3
13. 5
14. Virahānka–Fibonacci
15. 1
16. multiplying
17. addition
18. multiplication
19. 10
20. GP

True / False

1. False
2. True
3. True
4. True
5. True
6. False
7. False
8. True
9. True
10. True

End of Chapter