Comprehensive Solutions of Text book Exercises

of

Punjab State Education Board

Class IX

Mathematics

Prepared for Academic Excellence

by

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Exercise 1.1

Q.1. Is zero a rational number? Can you write it in the form p/q, where p and q are integers and $q \neq 0$?

Solution: Yes, zero is a rational number.

Explanation:

A rational number is any number that can be written in the form:

p / q
where p and q are integers and $q \neq 0$.

Zero can be written as:

$0 = 0 / 1$

Here:

- $p = 0$ (an integer)
- $q = 1$ (an integer and not equal to zero)

Q.2. Find six rational numbers between 3 and 4

Solution: Write 3 and 4 with a common denominator

$3 = 30 / 10$
$4 = 40 / 10$

Write rational numbers between them

Six rational numbers between 30 / 10 and 40 / 10 are:

31 / 10
32 / 10
33 / 10
34 / 10
35 / 10
36 / 10

Conclusion:

Six rational numbers between 3 and 4 are:

31/10, 32/10, 33/10, 34/10, 35/10, 36/10

Q.3. Find five rational numbers between 3/5 and 4/5

Solution: Convert fractions to same denominator

$3/5 = 30/50$
$4/5 = 40/50$

Write five rational numbers between them

Five rational numbers between 30/50 and 40/50 are:

31/50
32/50
33/50
34/50
35/50

Conclusion: Five rational numbers between 3/5 and 4/5 are:

31/50, 32/50, 33/50, 34/50, 35/50

Q.4. State whether the following statements are true or false. Give reasons.

(i) Every natural number is a whole number

Answer: True

Reason: Natural numbers are: 1, 2, 3, 4, ...

Whole numbers are: 0, 1, 2, 3, 4, ...

All natural numbers are included in whole numbers.

(ii) Every integer is a whole number

Answer: False

Reason: Integers include negative numbers:
..., −3, −2, −1, 0, 1, 2, 3, ...

Whole numbers do not include negative numbers.

(iii) Every rational number is a whole number

Answer: False

Reason: Rational numbers include fractions such as: $1/2, 3/4, 5/3$

Whole numbers do not include fractions.

End of Exercise

Exercise 1.2

Q.1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

Answer: True

Justification:
Real numbers include both rational and irrational numbers.

Examples:

- Rational numbers: $1/2, 3, -5$
- Irrational numbers: square root of 2, square root of 5, pi

Since irrational numbers are a part of the real number system, every irrational number is a real number.

✔ Statement is true.

(ii) Every point on the number line is of the form square root of m, where m is a natural number.

Answer: False

Justification:
Points on the number line represent all real numbers, such as:

- Rational numbers: $1/2, 3/4, -2$
- Irrational numbers: square root of 2, pi

But numbers like:

- $1/2$
- -3
- pi

cannot be written in the form square root of m, where m is a natural number.

✔ Statement is false.

(iii) Every real number is an irrational number.

Answer: False

Justification:
Real numbers include both rational and irrational numbers.

Examples of real but rational numbers:

- 2
- 5/3
- 0

Since rational numbers are real but not irrational, the statement is incorrect.

✔ Statement is false.

Q.2. Are the square roots of all positive integer's irrational? If not, give an example of the square root of a number that is a rational number.

Answer: No, the square roots of all positive integers are not irrational.

Explanation:

- Square roots of perfect squares are rational numbers.
- Square roots of non-perfect squares are irrational numbers.

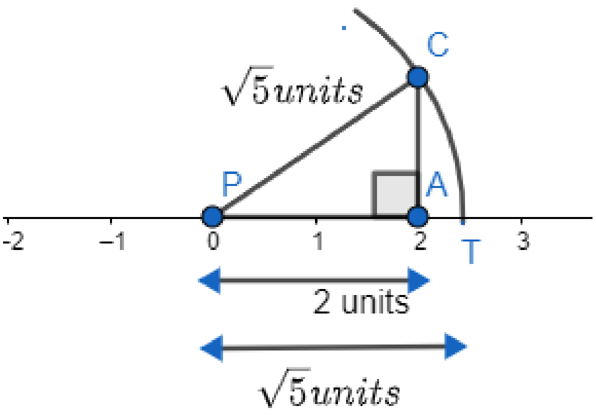
Example

Square root of 9 = 3
Square root of 16 = 4

Both 3 and 4 are rational numbers.

✔ Therefore, square roots of all positive integers are not irrational

Q.3. Show how square root of 5 can be represented on the number line



Steps of Construction

1. Draw a number line and mark point O (0).
2. Mark point A such that OA = 2 units.
3. From point A, draw a perpendicular AB of length 1 unit.
4. Join points O and B.
5. Using O as center and OB as radius, draw an arc cutting the number line at point P.

Explanation

- Triangle OAB is a right-angled triangle.
- By Pythagoras theorem:

$OB^2 = OA^2 + AB^2$
$OB^2 = 2^2 + 1^2$

$OB^2 = 4 + 1$
$OB^2 = 5$

$OB = \sqrt{5}$

Therefore, OP represents square root of 5 on the number line.

End of Exercise

Exercise 1.3

Q.1. Write the following in decimal form and say what kind of decimal expansion each has

(i) 36 / 100

36 / 100 = 0.36

Type: Terminating decimal
(Reason: Denominator has only factors 2 and 5)

(ii) 1 / 11

1 / 11 = 0.090909...

Type: Non-terminating recurring decimal
(Repeating block = 09)

(iii) 4 1/8

4 1/8 = 33 / 8
33 / 8 = 4.125

Type: Terminating decimal
(8 = 2 cubed)

(iv) 3 / 13

3 / 13 = 0.230769230769...

Type: Non-terminating recurring decimal
(Repeating block = 230769)

(v) 2 / 11

2 / 11 = 0.181818...

Type: Non-terminating recurring decimal
(Repeating block = 18)

(vi) 329 / 400

329 / 400 = 0.8225

Type: Terminating decimal
(400 = 2 to the 4 × 5 squared)

Q.2. You know that 1 / 7 = $0.\overline{142857}$. Can you predict the decimal expansions of 2/7, 3/7, 4/7, 5/7, 6/7 without division?

Observation

1 / 7 = 0.142857 (repeating block = 142857)

Pattern

Multiples of 1/7 give cyclic shifts of the same repeating block.

Fraction	Decimal expansion
2 / 7	0.285714...
3 / 7	0.428571...
4 / 7	0.571428...
5 / 7	0.714285...

Conclusion

Yes, we can predict them by observing the repeating remainders of 1/7.

Q.3. Express the following in the form p / q, where p and q are integers and q=0

(i) $0.\overline{6}$ with bar on 6 (0.666...)

Let x = 0.666...

10x = 6.666...

Subtract:
10x − x = 6
9x = 6
x = 6 / 9 = 2 / 3

(ii) $0.\overline{47}$ with bar on 47 (0.474747...)

Let x = 0.474747...

100x = 47.474747...

Subtract:
100x − x = 47
99x = 47
x = 47 / 99

(iii) $0.\overline{001}$

0.001 = 1 / 1000

Q.4. Express 0.9999... in the form p / q

Solution: Let x = 0.9999...

10x = 9.9999...

Subtract:
10x − x = 9

$9x = 9$
$x = 1$

Conclusion

0.9999... = 1

Not surprising, because the difference between them is zero

Q.5. What is the maximum number of digits in the repeating block of 1 / 17?

Solution: The maximum length of repeating digits of $1/q$ is $(q - 1)$.

For 1 / 17:
Maximum repeating digits = 16

Check

1 / 17 = 0.0588235294117647...

It has 16 repeating digits.

Q.6. What property must q satisfy for p / q to have a terminating decimal expansion?

After simplification, q must have only prime factors 2 and/or 5.

Examples

- 1 / 8 (8 = 2 cubed) → terminating
- 3 / 20 (20 = 2 squared × 5) → terminating
- 1 / 3 → non-terminating

Q.7. Write three numbers whose decimal expansions are non-terminating and non-recurring

Solution: Examples:

- square root of 2 = 1.414213...
- square root of 3 = 1.732050...
- pi = 3.141592...

(All are irrational numbers)

Q.8. Find three different irrational numbers between 5/7 and 9/11

Solution:

5 / 7 ≈ 0.714
9 / 11 ≈ 0.818

Three irrational numbers between them:

- square root of 0.55 ≈ 0.7416
- square root of 0.60 ≈ 0.7746
- square root of 0.65 ≈ 0.8062

Q.9. Classify the following as rational or irrational

(i) square root of 23

Irrational (23 is not a perfect square)

(ii) square root of 225

225 = 15 squared
So, square root of 225 = 15
Rational

(iii) 0.3796

Terminating decimal
Rational

(iv) 7.478478...

Repeating decimal
Rational

(v) 1.101001000100001...

Non-terminating, non-repeating
Irrational

End of Exercise

Exercise 1.4

Q.1. Classify the following numbers as rational or irrational. Give reasons.

(i) $2 - \sqrt{5}$

Reason:
$\sqrt{5}$ is an irrational number and 2 is a rational number.
The difference of a rational number and an irrational number is irrational.

$\therefore 2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

Solution:
$(3 + \sqrt{23}) - \sqrt{23}$
$= 3 + \sqrt{23} - \sqrt{23}$
$= 3$

Reason:
3 is an integer and hence a rational number.

$\therefore (3 + \sqrt{23}) - \sqrt{23}$ is a rational number.

(iii) $(2\sqrt{7}) / (7\sqrt{7})$

Solution:
$(2\sqrt{7}) / (7\sqrt{7})$
$= 2 / 7$ (by cancelling $\sqrt{7}$)

Reason:
$2/7$ is of the form p/q, where p and q are integers and $q \neq 0$.

$\therefore (2\sqrt{7}) / (7\sqrt{7})$ is a rational number.

(iv) $1 / \sqrt{2}$

Reason:
$\sqrt{2}$ is an irrational number.
Dividing a rational number by an irrational number gives an irrational number.

(Alternatively: $1/\sqrt{2} = \sqrt{2}/2$, which is irrational.)

$\therefore 1 / \sqrt{2}$ is an irrational number.

(v) 2π

Reason:
π is an irrational number and 2 is a non-zero rational number.
The product of a non-zero rational number and an irrational number is irrational.

$\therefore 2\pi$ is an irrational number.

Q.2. Simplify each of the following expressions

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$

Solution:
$(3 + \sqrt{3})(2 + \sqrt{2})$
$= 3\times2 + 3\times\sqrt{2} + \sqrt{3}\times2 + \sqrt{3}\times\sqrt{2}$
$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

Answer:
$6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$

Solution:
Using $(a + b)(a - b) = a^2 - b^2$

$(3 + \sqrt{3})\ (3 - \sqrt{3})$
$= 3^2 - (\sqrt{3})^2$
$= 9 - 3$
$= 6$

Answer: 6

(iii) $(\sqrt{5} + \sqrt{2})^2$

Solution:
$(\sqrt{5} + \sqrt{2})^2$
$= (\sqrt{5})^2 + 2(\sqrt{5})(\sqrt{2}) + (\sqrt{2})^2$
$= 5 + 2\sqrt{10} + 2$
$= 7 + 2\sqrt{10}$

Answer: $7 + 2\sqrt{10}$

(iv) $(\sqrt{5} - \sqrt{2})\ (\sqrt{5} + \sqrt{2})$

Solution:
Using $(a - b)\ (a + b) = a^2 - b^2$

$(\sqrt{5} - \sqrt{2})\ (\sqrt{5} + \sqrt{2})$
$= (\sqrt{5})^2 - (\sqrt{2})^2$
$= 5 - 2$
$= 3$

Answer: 3

Q.3. π is defined as $\pi = c / d$. This seems to contradict that π is irrational. Resolve this contradiction.

Solution: π is defined as the ratio of the circumference (c) of a circle to its diameter (d).

Although c and d are measurable lengths, their exact values cannot be written as integers.
The ratio c / d is not a ratio of two integers, hence it does not make π rational.

Conclusion

There is no contradiction.
π is irrational even though it is defined as a ratio of two lengths.

Q.4. Represent √9.3 on the number line

Steps of Construction

1. Draw a number line and mark point O (0).
2. Mark point A such that OA = 9.3 units.
3. From point A, draw a perpendicular AB of length 1 unit.
4. Join points O and B.
5. With O as centre and OB as radius, draw an arc to cut the number line at point P.

Explanation

In right-angled triangle OAB:
$OB^2 = OA^2 + AB^2$
$OB^2 = 9.3 + 1$
$OB^2 = 10.3$

Thus, OP represents √9.3 on the number line.

Q.5. Rationalize the denominators

(i) 1 / √7

Solution:
Multiply numerator and denominator by √7:

$1 / \sqrt{7} = \sqrt{7} / 7$

Answer: $\sqrt{7} / 7$

(ii) $1 / (\sqrt{7} - \sqrt{6})$

Solution: Multiply numerator and denominator by $(\sqrt{7} + \sqrt{6})$:

$1 / (\sqrt{7} - \sqrt{6})$
$= (\sqrt{7} + \sqrt{6}) / (7 - 6)$
$= \sqrt{7} + \sqrt{6}$

Answer: $\sqrt{7} + \sqrt{6}$

(iii) $1 / (\sqrt{5} + \sqrt{2})$

Solution: Multiply numerator and denominator by $(\sqrt{5} - \sqrt{2})$:

$1 / (\sqrt{5} + \sqrt{2})$
$= (\sqrt{5} - \sqrt{2}) / (5 - 2)$
$= (\sqrt{5} - \sqrt{2}) / 3$

Answer: $(\sqrt{5} - \sqrt{2}) / 3$

(iv) $1 / (\sqrt{7} - 2)$

Solution: Multiply numerator and denominator by $(\sqrt{7} + 2)$:

$1 / (\sqrt{7} - 2)$
$= (\sqrt{7} + 2) / (7 - 4)$
$= (\sqrt{7} + 2) / 3$

Answer: $(\sqrt{7} + 2) / 3$

End of Exercise

Exercise 1.5

Q.1. Find:

(i) $64^{1/2}$

Solution:
$64^{1/2} = \sqrt{64}$
$= 8$

Answer: 8

(ii) $32^{1/5}$

Solution:
$32 = 2^5$

$32^{1/5} = (2^5)^{1/5}$
$= 2$

Answer: 2

(iii) $125^{1/3}$

Solution:
$125 = 5^3$

$125^{1/3} = (5^3)^{1/3}$
$= 5$

Answer: 5

Q.2. Find:

(i) $9^{3/2}$

Solution:
$9^{3/2} = (\sqrt{9})^{3}$
$= 3^3$
$= 27$

Answer: 27

(ii) $32^{2/5}$

Solution:
$32 = 2^5$

$32^{2/5} = (2^5)^{2/5}$
$= 2^2$
$= 4$

Answer: 4

(iii) $16^{3/4}$

Solution:
$16 = 2^4$

$16^{3/4} = (2^4)^{3/4}$
$= 2^3$
$= 8$

Answer: 8

(iv) $125^{-1/3}$

Solution:
$125 = 5^3$

$125^{-1/3} = (5^3)^{-1/3}$
$= 5^{-1}$
$= 1/5$

Answer: $1/5$

Q.3. Simplify:

(i) $2^{2/3} \times 2^{1/5}$

Solution: Using the law: $a^m \times a^n = a^{m+n}$

$2^{2/3} \times 2^{1/5}$
$= 2^{2/3 + 1/5}$

LCM of 3 and 5 = 15

$2/3 = 10/15$
$1/5 = 3/15$

$= 2^{13/15}$

Answer: $2^{13/15}$

(ii) $(1/3^3)^7$

Solution: $(1/3^3)^7$
$= 1^7 / 3^{21}$
$= 1 / 3^{21}$

Answer: $1/3^{21}$

(iii) $11^{1/2} / 11^{1/4}$

Solution:
Using the law: $a^m / a^n = a^{m-n}$

$11^{1/2} / 11^{1/4}$
$= 11^{1/4}$

$11^{1/4} = \sqrt{\sqrt{11}}$

Answer: $\sqrt{\sqrt{11}}$

(iv) $7^{1/2} \times 8^{1/2}$

Solution:
$7^{1/2} \times 8^{1/2}$
$= \sqrt{7} \times \sqrt{8}$
$= \sqrt{56}$
$= 2\sqrt{14}$

Answer: $2\sqrt{14}$

End of Chapter

Exercise 2.1

Q.1.Which of the following expressions are polynomials in one variable and which are not? Give reasons.

(i) $4x^2 - 3x + 7$

Yes, Polynomial in one variable (x)
Reason: Powers of x are whole numbers and coefficients are real.

(ii) $y^2 + \sqrt{2}$

Yes, Polynomial in one variable (y)
Reason: $\sqrt{2}$ is a real constant and the power of y is a whole number.

(iii) $3\sqrt{t} + t\sqrt{2}$

Not a polynomial
Reason: $\sqrt{t} = t^{1/2}$, power of variable is not a whole number.

(iv) $y + 2/y$

Not a polynomial
Reason: Variable y occurs in the denominator (negative power).

(v) $x^{10} + y^3 + t^{50}$

Not a polynomial in one variable
Reason: It contains more than one variable.

Q.2 Write the coefficient of x^2 in each of the following:

(i) $2 + x^2 + x$

Coefficient of $x^2 = 1$

(ii) $2 - x^2 + x^3$

Coefficient of $x^2 = -1$

(iii) $(\pi/2)\, x^2 + x$

Coefficient of $x^2 = \pi/2$

(iv) $\sqrt{2}x - 1$

Coefficient of $x^2 = 0$
(There is no x^2 term.)

Q. 3 Give one example each of:

- Binomial of degree 35: $x^{35} + 1$
- Monomial of degree 100: x^{100}

Q.4 Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$

Degree $= 3$

(ii) $4 - y^2$

Degree $= 2$

(iii) $5t - \sqrt{7}$

Degree = 1

(iv) 3

Degree = 0 (Constant polynomial)

Q. 5 Classify the following as linear, quadratic or cubic polynomials:

(i) $x^2 + x$ ➞ Quadratic polynomial

(ii) $x - x^3$ ➞ Cubic polynomial

(iii) $y + y^2 + 4$ ➞ Quadratic polynomial

(iv) $1 + x$ ➞ Linear polynomial

(v) $3t$ ➞ Linear polynomial

(vi) r^2 ➞ Quadratic polynomial

(vii) $7x^3$ ➞ Cubic polynomial

End of Exercise

Exercise 2.2

Q.1. Find the value of the polynomial $p(x) = 5x - 4x^2 + 3$

(i) $x = 0$

$p\ (0) = 5(0) - 4(0)^2 + 3$
$p\ (0) = 3$

(ii) $x = -1$

$P\ (-1) = 5(-1) - 4(-1)^2 + 3$
$p\ (-1) = -5 - 4 + 3$
$p\ (-1) = -6$

(iii) $x = 2$

$p\ (2) = 5(2) - 4(2)^2 + 3$
$p\ (2) = 10 - 16 + 3$
$p\ (2) = -3$

Q. 2. Find p (0), p (1) and p (2)

(i) $p(y) = y^2 - y + 1$

$p\ (0) = (0)^2 - 0 + 1 = 1$
$p\ (1) = (1)^2 - 1 + 1 = 1$
$p\ (2) = (2)^2 - 2 + 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$p\ (0) = 2$

$p(1) = 2 + 1 + 2 - 1 = 4$
$p(2) = 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$

$p(0) = 0$
$p(1) = 1$
$p(2) = 8$

(iv) $p(x) = (x - 1)(x + 1)$

$P(0) = (-1)(1) = -1$
$p(1) = (0)(2) = 0$
$p(2) = (1)(3) = 3$

Q. 3. Verify whether the following are zeroes of the polynomial

(i) $p(x) = 3x + 1, x = -1/3$

Substitute $x = -1/3$ in $p(x)$:

$p(-1/3) = 3(-1/3) + 1$
$p(-1/3) = -1 + 1$
$p(-1/3) = 0$

Conclusion: $-1/3$ is a zero of the polynomials.

(ii) $p(x) = 5x - \pi, x = 4/5$

Substitute $x = 4/5$:

$p(4/5) = 5(4/5) - \pi$
$p(4/5) = 4 - \pi$

Since $\pi \neq 4$,
$4 - \pi \neq 0$

Conclusion: 4/5 is not a zero of the polynomials.

(iii) $p(x) = x^2 - 1, x = 1, -1$

For $x = 1$:

$p\ (1) = (1)\ ^2 - 1$
$p\ (1) = 1 - 1 = 0$

For $x = -1$:

$p\ (-1) = (-1)\ ^2 - 1$
$p\ (-1) = 1 - 1 = 0$

Conclusion: 1 and -1 are zeroes of the polynomial.

(iv) $p(x) = (x + 1)\ (x - 2), x = -1, 2$

For $x = -1$:

$p\ (-1) = (-1 + 1)\ (-1 - 2)$
$p\ (-1) = 0$

For $x = 2$:

$p\ (2) = (2 + 1)\ (2 - 2)$
$p\ (2) = 0$

Conclusion: -1 and 2 are zeroes of the polynomial.

(v) $p(x) = x^2, x = 0$

$p\ (0) = (0)\ ^2 = 0$

Conclusion: 0 is a zero of the polynomials.

(vi) $p(x) = lx + m, x = -m/l$

Substitute $x = -m/l$:

$p(-m/l) = l(-m/l) + m$
$p(-m/l) = -m + m$
$p(-m/l) = 0$

Conclusion: $-m/l$ is a zero of the polynomials.

(vii) $p(x) = 3x^2 - 1, x = -1/\sqrt{3}, 1/\sqrt{3}$

For $x = 1/\sqrt{3}$:

$p\,(1/\sqrt{3}) = 3(1/3) - 1$
$p\,(1/\sqrt{3}) = 1 - 1 = 0$

For $x = -1/\sqrt{3}$:

$p\,(-1/\sqrt{3}) = 3(1/3) - 1$
$p\,(-1/\sqrt{3}) = 1 - 1 = 0$

Conclusion: both values are zeroes of the polynomial.

(viii) $p(x) = 2x + 1, x = 1/2$

$p\,(1/2) = 2(1/2) + 1$
$p\,(1/2) = 2$

Since $2 \neq 0$,

Conclusion: $1/2$ is not a zero of the polynomials.

Q.4. Find the zero of the polynomials

(i) $p(x) = x + 5$

$x + 5 = 0$
$x = -5$

$\text{Zero} = -5$

(ii) $p(x) = x - 5$

$x - 5 = 0$
$x = 5$

$\text{Zero} = 5$

(iii) $p(x) = 2x + 5$

$2x + 5 = 0$
$2x = -5$
$x = -5/2$

$\text{Zero} = -5/2$

(iv) $p(x) = 3x - 2$

$3x - 2 = 0$
$3x = 2$
$x = 2/3$

$\text{Zero} = 2/3$

(v) $p(x) = 3x$

$3x = 0$
$x = 0$

$\text{Zero} = 0$

(vi) $p(x) = ax, a \neq 0$

$ax = 0$
$x = 0$

Zero = 0

(vii) $p(x) = cx + d, c \neq 0$

$cx + d = 0$
$cx = -d$
$x = -d/c$

Zero = $-d/c$

End of Exercise

Exercise 2.3

Q. 1. Determine which of the following polynomials has $(x + 1)$ as a factor

(i) $p(x) = x^3 + x^2 + x + 1$

According to the Factor Theorem,
$(x + 1)$ is a factor of $p(x)$ if $p(-1) = 0$.

$$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$$
$$= -1 + 1 - 1 + 1$$
$$= 0$$

∴ $(x + 1)$ is a factor of $x^3 + x^2 + x + 1$.

(ii) $p(x) = x^4 + x^3 + x^2 + x + 1$

$$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$
$$= 1 - 1 + 1 - 1 + 1$$
$$= 1 \neq 0$$

∴ $(x + 1)$ is not a factor of the given polynomial.

(iii) $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$
$$= 1 - 3 + 3 - 1 + 1$$
$$= 1 \neq 0$$

∴ $(x + 1)$ is not a factor of the given polynomial.

(iv) $p(x) = x^3 - x^2 - (2 + \sqrt{2})\, x + \sqrt{2}$

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$
$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$
$= 0$

$\therefore$ (x + 1) is a factor of the given polynomial.

Q. 2. Use the Factor Theorem to determine whether g(x) is a factor of p(x)

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

Since $g(x) = x + 1$, we find p (−1).

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$
$= -2 + 1 + 2 - 1$
$= 0$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

Since $g(x) = x + 2$, we check p (−2).

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$
$= -8 + 12 - 6 + 1$
$= -1 \neq 0$

$\therefore$ (x + 2) is not a factor of p(x).

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

Since $g(x) = x - 3$, we check p (3).

$p(3) = (3)^3 - 4(3)^2 + 3 + 6$
$= 27 - 36 + 3 + 6$
$= 0$

$\therefore$ (x − 3) is a factor of p(x).

Q. 3. Find the value of k, if $(x - 1)$ is a factor of $p(x)$

(i) $p(x) = x^2 + x + k$

$p(1) = (1)^2 + 1 + k$
$p(1) = 2 + k$

Since $p(1) = 0$,
$2 + k = 0$
$k = -2$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

$p(1) = 2(1)^2 + k(1) + \sqrt{2}$
$p(1) = 2 + k + \sqrt{2}$

$2 + k + \sqrt{2} = 0$
$k = -2 - \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}\,x + 1$

$p(1) = k - \sqrt{2} + 1$

$k - \sqrt{2} + 1 = 0$
$k = \sqrt{2} - 1$

(iv) $p(x) = kx^2 - 3x + k$

$p(1) = k - 3 + k$
$p(1) = 2k - 3$

$2k - 3 = 0$
$k = 3/2$

Q. 4. Factorize

(i) $12x^2 - 7x + 1$

Product = $12 \times 1 = 12$
Middle term = $-7x$

$12x^2 - 4x - 3x + 1$
$= 4x(3x - 1) - 1(3x - 1)$
$= (4x - 1)(3x - 1)$

(ii) $2x^2 + 7x + 3$

Product = $2 \times 3 = 6$
Middle term = $7x$

$2x^2 + 6x + x + 3$
$= 2x(x + 3) + 1(x + 3)$
$= (2x + 1)(x + 3)$

(iii) $6x^2 + 5x - 6$

Product = $6 \times (-6) = -36$
Middle term = $5x$

$6x^2 + 9x - 4x - 6$
$= 3x(2x + 3) - 2(2x + 3)$
$= (3x - 2)(2x + 3)$

(iv) $3x^2 - x - 4$

Product = $3 \times (-4) = -12$
Middle term = $-x$

$3x^2 - 4x + 3x - 4$
$= x\,(3x - 4) + 1(3x - 4)$
$= (x + 1)\,(3x - 4)$

Q. 5. Factorize

(i) $x^3 - 2x^2 - x + 2$

$= (x^3 - 2x^2) - (x - 2)$
$= x^2\,(x - 2) - 1(x - 2)$
$= (x^2 - 1)\,(x - 2)$
$= (x - 1)\,(x + 1)\,(x - 2)$

(ii) $x^3 - 3x^2 - 9x - 5$

Try $x = 5$:

$p(5) = 125 - 75 - 45 - 5 = 0$

$\therefore$ $(x - 5)$ is a factor.

Dividing,
$= (x - 5)\,(x^2 + 2x + 1)$
$= (x - 5)\,(x + 1)^2$

(iii) $x^3 + 13x^2 + 32x + 20$

Try $x = -1$:

$p(-1) = -1 + 13 - 32 + 20 = 0$

$\therefore$ $(x + 1)$ is a factor.

Dividing,
$= (x + 1)\ (x^2 + 12x + 20)$
$= (x + 1)\ (x + 10)\ (x + 2)$

(iv) $2y^3 + y^2 - 2y - 1$

$= (2y^3 + y^2) - (2y + 1)$
$= y^2(2y + 1) - 1(2y + 1)$
$= (y^2 - 1)\ (2y + 1)$
$= (y - 1)\ (y + 1)\ 1(2y + 1)$

End of Exercise

Exercise 2.4

Q.1. Use suitable identities to find the following products

(i) $(x + 4)(x + 10)$

Using the identity:
$(a + b)(a + c) = a^2 + a(b + c) + bc$

Here, $a = x, b = 4, c = 10$

$(x + 4)(x + 10)$
$= x^2 + x(4 + 10) + (4)(10)$
$= x^2 + 14x + 40$

Answer: $x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$

Using the identity:
$(x + a)(x + b) = x^2 + x(a + b) + ab$

Here, $a = 8, b = -10$

$(x + 8)(x - 10)$
$= x^2 + x(8 - 10) + (8)(-10)$
$= x^2 - 2x - 80$

Answer: $x^2 - 2x - 80$

(iii) $(3x + 4)(3x - 5)$

Using the identity:
$(a + b)(a - c) = a^2 + a(b - c) - bc$

Here, $a = 3x, b = 4, c = 5$

$(3x + 4)(3x - 5)$
$= (3x)^2 + 3x(4 - 5) - (4)(5)$
$= 9x^2 - 3x - 20$

Answer: $9x^2 - 3x - 20$

(iv) $(y^2 + 3/2)(y^2 - 3/2)$

Using the identity:
$(a + b)(a - b) = a^2 - b^2$

Here, $a = y^2$, $b = 3/2$

$(y^2 + 3/2)(y^2 - 3/2)$
$= (y^2)^2 - (3/2)^2$
$= y^4 - 9/4$

Answer: $y^4 - 9/4$

(v) $(3 - 2x)(3 + 2x)$

Using the identity:
$(a - b)(a + b) = a^2 - b^2$

Here, $a = 3$, $b = 2x$

$(3 - 2x)(3 + 2x)$
$= 3^2 - (2x)^2$
$= 9 - 4x^2$

Answer: $9 - 4x^2$

Q. 2. Evaluate the following products without multiplying directly

(i) 103×107

Using the identity:
$(a - b)(a + b) = a^2 - b^2$

$103 = 105 - 2$
$107 = 105 + 2$

103×107
$= (105 - 2)(105 + 2)$
$= 105^2 - 2^2$
$= 11025 - 4$
$= 11021$

Answer: 11021

(ii) 95×96

Let $95 = 100 - 5$ and $96 = 100 - 4$

Using the identity:
$(a - b)(a - c) = a^2 - a(b + c) + bc$

95×96
$= (100 - 5)(100 - 4)$
$= 100^2 - 100(5 + 4) + (5)(4)$
$= 10000 - 900 + 20$
$= 9120$

Answer: 9120

(iii) 104×96

Using the identity:
$(a + b)(a - b) = a^2 - b^2$

$104 = 100 + 4$
$96 = 100 - 4$

104×96
$= (100 + 4)(100 - 4)$
$= 100^2 - 4^2$

$= 10000 - 16$
$= 9984$

Answer: 9984

Q. 3. Factorize the following using appropriate identities

(i) $9x^2 + 6xy + y^2$

Using the identity:
$a^2 + 2ab + b^2 = (a + b)^2$

Here, $a = 3x$, $b = y$

$9x^2 + 6xy + y^2$
$= (3x)^2 + 2(3x)(y) + y^2$
$= (3x + y)^2$

Answer: $(3x + y)^2$

(ii) $4y^2 - 4y + 1$

Using the identity:
$a^2 - 2ab + b^2 = (a - b)^2$

Here, $a = 2y$, $b = 1$

$4y^2 - 4y + 1$
$= (2y)^2 - 2(2y)(1) + 1^2$
$= (2y - 1)^2$

Answer: $(2y - 1)^2$

(iii) $x^2 - y^2/100$

Using the identity:
$a^2 - b^2 = (a + b)(a - b)$

Here, $a = x$, $b = y/10$

$x^2 - y^2/100$
$= x^2 - (y/10)^2$
$= (x + y/10)(x - y/10)$

Answer: $(x + y/10)(x - y/10)$

Q. 4. Expand each of the following using suitable identities

(i) $(x + 2y + 4z)^2$

Using identity:
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = x$, $b = 2y$, $c = 4z$

$(x + 2y + 4z)^2$
$= x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$
$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

Answer: $x^2 + 4y^2 + 16z^2 + 4xy + 8xz + 16yz$

(ii) $(2x - y + z)^2$

Using identity $(a + b + c)^2$

Here, $a = 2x$, $b = -y$, $c = z$

$= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$
$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

Answer: $4x^2 + y^2 + z^2 - 4xy + 4xz - 2yz$

(iii) $(-2x + 3y + 2z)^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$
$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

Answer: $4x^2 + 9y^2 + 4z^2 - 12xy - 8xz + 12yz$

(iv) $(3a - 7b - c)^2$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$
$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$

Answer: $9a^2 + 49b^2 + c^2 - 42ab - 6ac + 14bc$

(v) $(-2x + 5y - 3z)^2$

$= 4x^2 + 25y^2 + 9z^2 - 20xy + 12xz - 30yz$

Answer: $4x^2 + 25y^2 + 9z^2 - 20xy + 12xz - 30yz$

(vi) $[1/4\, a - 1/2\, b + 1]^2$

Let $a = a/4$, $b = -b/2$, $c = 1$

$= (a/4)^2 + (b/2)^2 + 1^2 + 2$

Q.5. Factorize

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Rearranging the terms:

$= 4x^2 + 9y^2 + 16z^2 + 12xy - 16xz - 24yz$

Observe that:

$4x^2 = (2x)^2$
$9y^2 = (3y)^2$
$16z^2 = (4z)^2$

And the middle terms are:
$2(2x)(3y) = 12xy$
$-2(2x)(4z) = -16xz$
$-2(3y)(4z) = -24yz$

Using identity:
$a^2 + b^2 + c^2 + 2ab - 2ac - 2bc = (a + b - c)^2$

Here, $a = 2x, b = 3y, c = 4z$

∴ Factorized form:
$(2x + 3y - 4z)^2$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

Rewrite terms:

$2x^2 = (\sqrt{2}\,x)^2$
$y^2 = y^2$
$8z^2 = (2\sqrt{2}\,z)^2$

Middle terms:
$-2(\sqrt{2}\,x)(y) = -2\sqrt{2}\,xy$
$-2(\sqrt{2}\,x)(2\sqrt{2}\,z) = -8xz$
$2(y)(2\sqrt{2}\,z) = 4\sqrt{2}\,yz$

Using identity:
$a^2 + b^2 + c^2 - 2ab - 2ac + 2bc = (a - b - c)^2$

Here, $a = \sqrt{2}\,x, b = y, c = 2\sqrt{2}\,z$

∴ Factorized form:
$(\sqrt{2}\,x - y - 2\sqrt{2}\,z)^2$

Q.6. Write the following cubes in expanded form

(i) $(2x + 1)^3$

Using identity:
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Here, a = 2x, b = 1

$= (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + 1$
$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3$

Using identity:
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Here, a = 2a, b = 3b

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(iii) $(3/2\,x + 1)^3$

Let a = 3/2 x, b = 1

$= (3/2\,x)^3 + 3(3/2\,x)^2(1) + 3(3/2\,x)(1)^2 + 1$
$= 27/8\,x^3 + 27/4\,x^2 + 9/2\,x + 1$

(iv) $(x - 2/3\,y)^3$

Using $(a - b)^3$

$= x^3 - 3x^2(2/3\,y) + 3x(2/3\,y)^2 - (2/3\,y)^3$
$= x^3 - 2x^2y + 4/3\,xy^2 - 8/27\,y^3$

Q. 7. Evaluate using suitable identities

(i) $(99)^3$

$99 = 100 - 1$

$(99)^3 = (100 - 1)^3$
$= 100^3 - 3 \cdot 100^2 \cdot 1 + 3 \cdot 100 \cdot 1^2 - 1$
$= 1000000 - 30000 + 300 - 1$
$= 970299$

(ii) $(102)^3$

$102 = 100 + 2$

$= (100 + 2)^3$
$= 100^3 + 3 \cdot 100^2 \cdot 2 + 3 \cdot 100 \cdot 4 + 8$
$= 1061208$

(iii) $(998)^3$

$998 = 1000 - 2$

$= (1000 - 2)^3$
$= 1000^3 - 3 \cdot 1000^2 \cdot 2 + 3 \cdot 1000 \cdot 4 - 8$
$= 994011992$

Q. 8. Factorize each of the following

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

Rearranging:

$= 8a^3 + 12a^2b + 6ab^2 + b^3$

Using identity:
$a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$

Here, $a = 2a, b = b$

$\therefore (2a + b)^3$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

Rearranging:

$= 8a^3 - 12a^2b + 6ab^2 - b^3$

Using $(a - b)^3$

$\therefore (2a - b)^3$

(iii) $27 - 125a^3 - 135a + 225a^2$

Rearranging:

$= 27 - 135a + 225a^2 - 125a^3$

$= (3 - 5a)^3$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

Rearranging:

$= 64a^3 - 144a^2b + 108ab^2 - 27b^3$

$= (4a - 3b)^3$

(v) $27p^3 - 1/216 - 9/2\ p^2 + 1/4\ p$

Rewrite as:

$= (3p)^3 - (1/6)^3 - 3(3p)^2 (1/6) + 3(3p) (1/6)^2$

Using $(a - b)^3$

$\therefore (3p - 1/6)^3$

Q. 9. Verify

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

RHS $= (x + y)(x^2 - xy + y^2)$
$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$
$= x^3 + y^3 =$ LHS

Verified

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

RHS $= (x - y)(x^2 + xy + y^2)$
$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$
$= x^3 - y^3 =$ LHS

Verified

Q. 10. Factorize

(i) $27y^3 + 125z^3$

$= (3y)^3 + (5z)^3$

Using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$\therefore (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) $64m^3 - 343n^3$

$= (4m)^3 - (7n)^3$

Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$\therefore (4m - 7n)(16m^2 + 28mn + 49n^2)$

Q. 11. Factorize: $27x^3 + y^3 + z^3 - 9xyz$

We use the identity:

$a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Here,
$a = 3x, b = y, c = z$

Substituting,

$27x^3 + y^3 + z^3 - 9xyz$
$= (3x + y + z)[(3x)^2 + y^2 + z^2 - 3xy - yz - 3xz]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - 3xz - yz)$

Answer:
$(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - 3xz - yz)$

Q. 12. Verify that

$x^3 + y^3 + z^3 - 3xyz = 1/2\,(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

LHS

$x^3 + y^3 + z^3 - 3xyz$

Using identity:

$x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

RHS

$1/2\,(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Expand the bracket:

$(x - y)^2 = x^2 - 2xy + y^2$
$(y - z)^2 = y^2 - 2yz + z^2$
$(z - x)^2 = z^2 - 2zx + x^2$

Adding,

$= 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$

Factor out 2:

$= 2(x^2 + y^2 + z^2 - xy - yz - zx)$

Now multiply:

$RHS = 1/2\ (x + y + z) \cdot 2(x^2 + y^2 + z^2 - xy - yz - zx)$
$RHS = (x + y + z)\ (x^2 + y^2 + z^2 - xy - yz - zx)$

Conclusion

LHS = RHS

Hence verified.

Q. 13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

Using identity:

$x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)\ (x^2 + y^2 + z^2 - xy - yz - zx)$

Given: $x + y + z = 0$

Substitute:

$x^3 + y^3 + z^3 - 3xyz = 0$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$

Hence proved.

Q. 14. Without actually calculating the cubes, find the value

(i) $(-12)^3 + (7)^3 + (5)^3$

Here,
$-12 + 7 + 5 = 0$

Using identity:

$a^3 + b^3 + c^3 = 3abc$, when $a + b + c = 0$

So,

$(-12)^3 + 7^3 + 5^3$
$= 3(-12)(7)(5)$
$= -1260$

Answer: -1260

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Check sum:

$28 - 15 - 13 = 0$

Using the same identity:

$= 3(28)(-15)(-13)$
$= 16380$

Answer: 16380

Q. 15. Give possible expressions for the length and breadth of rectangles

(i) Area $= 25a^2 - 35a + 12$

Factorize:

$25a^2 - 35a + 12$

Product $= 25 \times 12 = 300$
Middle term $= -35a$

Split middle term:

$25a^2 - 15a - 20a + 12$
$= 5a(5a - 3) - 4(5a - 3)$
$= (5a - 3)(5a - 4)$

Possible length and breadth:
$5a - 3$ and $5a - 4$

(ii) Area $= 35y^2 + 13y - 12$

Factorize:

$35y^2 + 13y - 12$

Product $= 35 \times (-12) = -420$
Middle term $= 13y$

Split middle term:

$35y^2 + 28y - 15y - 12$
$= 7y\,(5y + 4) - 3(5y + 4)$
$= (7y - 3)\,(5y + 4)$

Possible length and breadth:
$7y - 3$ and $5y + 4$

Q. 16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume $= 3x^2 - 12x$

Factorize the given expression:

$3x^2 - 12x$
$= 3x\ (x - 4)$

Now express it as a product of three factors:

$3x\ (x - 4) = 3 \cdot x \cdot (x - 4)$

Possible dimensions of the cuboid:

Length $= 3$
Breadth $= x$
Height $= x - 4$

(Any rearrangement of these factors is also acceptable.)

(ii) Volume $= 12ky^2 + 8ky - 20k$

Factorize the given expression:

$12ky^2 + 8ky - 20k$

First take common factor k:

$= k\ (12y^2 + 8y - 20)$

Now factories the quadratic expression:

$12y^2 + 8y - 20$

Product $= 12 \times (-20) = -240$
Middle term $= 8y$

Split the middle term:

$12y^2 + 20y - 12y - 20$
$= 4y\ (3y + 5) - 4(3y + 5)$
$= (4y - 4)\ (3y + 5)$

So,

Volume = k (4y – 4) (3y + 5)

Factor out 4 from (4y – 4):

= 4k (y – 1) (3y + 5)

Possible dimensions of the cuboid:

Length = 4k
Breadth = y – 1
Height = 3y + 5

End of chapter

Exercise 3.1

Q. 1. How will you describe the position of a table lamp on your study table to another person?

To describe the position of a table lamp, we use reference points.

For example, we can say:

- The table lamp is to the right of the book, or
- The lamp is near the back edge of the table, or
- The lamp is between the notebook and the pen stand.

Thus, the position of the table lamp is described relative to other objects on the table.

Q. 2. (Street Plan) A city has two main roads which cross each other at the center of the city. These two roads are along the North–South direction and East–West direction.

Given (Street Plan)

- Two main roads cross at the centre of the city.
  - One runs North–South
  - One runs East–West
- All other streets are parallel to these two roads.
- Distance between consecutive streets = 200 m
- There are 5 streets in each direction.
- A crossing is named as (m, n) if:
  - the m^{th} North–South Street and
  - the n^{th} East–West Street intersect.

Understanding the Convention

- North–South streets are numbered 1 to 5
- East–West streets are numbered 1 to 5
- A cross-street (m, n) means:
  - m choices from North–South streets
  - n choices from East–West streets

(i) How many cross-streets can be referred to as (4, 3)?

- Choose 4 North–South streets from 5
- Choose 3 East–West streets from 5

Number of ways:

$$4 \times 3 = 12$$

Answer: 12

(ii) How many cross-streets can be referred to as (3, 4)?

- Choose 3 North–South streets from 5
- Choose 4 East–West streets from 5

Number of ways:

$$3 \times 4 = 12$$

Answer: 12

End of Exercise

Exercise 3.2

Q.1. Write the answer of each of the following questions:

(i) What is the name of the horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

- The horizontal line is called the X-axis.
- The vertical line is called the Y-axis.

(ii) What is the name of each part of the plane formed by these two lines?

The two axes divide the plane into four parts, called quadrants.

They are:

1. First Quadrant
2. Second Quadrant
3. Third Quadrant
4. Fourth Quadrant

(iii) Write the name of the point where these two lines intersect.

The point where the X-axis and Y-axis intersect is called the origin.

It is denoted by O (0, 0).

Q.2 See Fig. 3.14 and write the following:

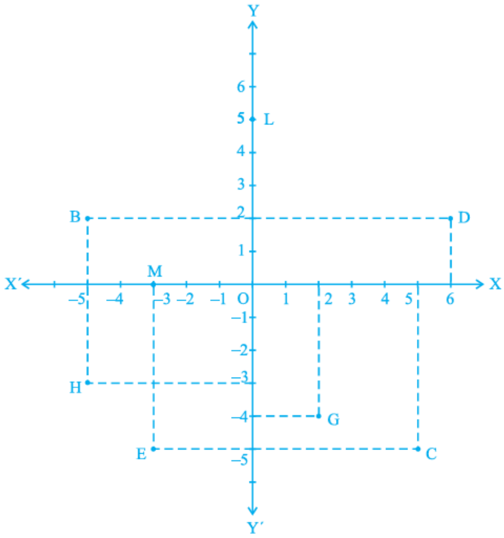


Fig. 3.14

(i) The coordinates of B

From the figure:

- Point B lies 5 units left of the origin → x = –5
- Point B lies 2 units above the x-axis → y = 2

B= (–5,2)

(ii) The coordinates of C

From the figure:

- Point C lies 6 units right of the origin → x = 6
- Point C lies 5 units below the x-axis → y = –5
- B= (6, –5)

(iii) The point identified by the coordinates (–3, –5)

From the figure:

- x = –3 → 3 units to the left of the origin
- y = –5 → 5 units below the x-axis

This point is labelled E in the diagram.

E= (–3, –5)

(iv) The point identified by the coordinates (2, —4)

- Move 2 units right from the origin
- Move 4 units down

This point is labelled G = $(2, -4)$

(v) The abscissa of the point D

- Abscissa = x-coordinate

From the figure, point D lies at x = 6.

$$\boxed{\text{Abscissa of } D = 6}$$

(vi) The ordinate of the point H

- Ordinate = y-coordinate

From the figure, point H lies at y = -3.

$$\boxed{\text{Ordinate of } H = -3}$$

(vii) The coordinates of the point L

From the figure:

- x = 0 (on the Y-axis)
- y = 5

L = (0,5)

(viii) The coordinates of the point M

From the figure:

- x = -3
- y = 1

M= (-3,1)

End of chapter

Exercise 4.1

Q.1. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

Let
Cost of a notebook = ₹ x
Cost of a pen = ₹ y

According to the given statement:
Cost of notebook = 2 × cost of pen

So,
$x = 2y$

Rewriting in linear equation form:
$x - 2y = 0$

Required linear equation:
$x - 2y = 0$

Q. 2. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case.

(i) $2x + 3y = 9.3\overline{5}$

Bringing all terms to LHS:

$2x + 3y - 9.35 = 0$

Comparing with $ax + by + c = 0$:

$a = 2, \quad b = 3, \quad c = -9.35$

(ii) $x - y/5 - 10 = 0$

Multiply the whole equation by 5 to remove the fraction:

$5x - y - 50 = 0$

So,

$a = 5, \quad b = -1, \quad c = -50$

(iii) $-2x + 3y = 6$

Bring all terms to LHS:

$-2x + 3y - 6 = 0$

So,

$a = -2, \quad b = 3, \quad c = -6$

(iv) $x = 3y$

Bring all terms to one side:

$x - 3y = 0$

So,

$a = 1, \quad b = -3, \quad c = 0$

(v) $2x = -5y$

Bring all terms to one side:

$2x + 5y = 0$

So,

$a = 2, \quad b = 5, \quad c = 0$

(vi) $3x + 2 = 0$

Rewrite as:

$3x + 0y + 2 = 0$

So,

$a = 3, \quad b = 0, \quad c = 2$

(vii) $y - 2 = 0$

Rewrite as:

$0x + y - 2 = 0$

So,

$a = 0, \quad b = 1, \quad c = -2$

(viii) $5 = 2x$

Bring all terms to LHS:

$2x - 5 = 0$

So,

$a = 2, \quad b = 0, \quad c = -5$

End of Exercise

Exercise 4.2

Q.1. Which one of the following options is true, and why?

$y = 3x + 5$ has:
(i) a unique solution,
(ii) only two solutions,
(iii) infinitely many solutions.

Given equation:
$y = 3x + 5$

This is a linear equation in two variables.

A linear equation in two variables has infinitely many solutions, because for every value of x, there is a corresponding value of y.

∴ Correct option:
(iii) Infinitely many solutions

Q.2. Write four solutions for each of the following equations

(i) $2x + y = 7$

Rewrite the equation:

$y = 7 - 2x$

Now choose different values of x:

x	0	1	2	3
y	7	5	3	2

Four solutions:
(0, 7), (1, 5), (2, 3), (3, 1)

(ii) $\pi x + y = 9$

Rewrite the equation:

$y = 9 - \pi x$

Choose different values of x:

x	0	1	2	-1
y	9	9 - π	9 - 2 π	9 + π

Four solutions:
$(0, 9), (1, 9 - \pi), (2, 9 - 2\pi), (-1, 9 + \pi)$

(iii) $x = 4y$

Rewrite as:

$x = 4y$

Choose values of y:

x	0	4	8	-4
y	0	1	2	-1

Four solutions:
$(0, 0), (4, 1), (8, 2), (-4, -1)$

Q.3. Check which of the following are solutions of the equation $x - 2y = 4$

Substitute each ordered pair in the equation.

(i) (0, 2)

$\text{LHS} = 0 - 2(2) = -4 \neq 4$

✗ Not a solution

(ii) $(2, 0)$

$\text{LHS} = 2 - 0 = 2 \neq 4$

✗ Not a solution

(iii) $(4, 0)$

$\text{LHS} = 4 - 0 = 4 = \text{RHS}$

☑ Solution

(iv) $(\sqrt{2}, 4\sqrt{2})$

$\text{LHS} = \sqrt{2} - 2(4\sqrt{2})$
$= \sqrt{2} - 8\sqrt{2}$
$= -7\sqrt{2} \neq 4$

✗ Not a solution

(v) $(1, 1)$

$\text{LHS} = 1 - 2(1) = -1 \neq 4$

✗ Not a solution

Conclusion (Q3): Only $(4, 0)$ is a solution of the equation $x - 2y = 4$.

Q.4. Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$

Substitute $x = 2$ and $y = 1$:

$2x + 3y = 2(2) + 3(1)$
$= 4 + 3$
$= 7$

$\therefore k = 7$

End of Chapter

Exercise 5.1

Q.1. Which of the following statements are true and which are false? Give reasons for your answers.

(i) Only one line can pass through a single point.
False.
Reason: Infinitely many lines can be drawn passing through a single point in different directions.

(ii) There are an infinite number of lines which pass through two distinct points.
False.
Reason: Through two distinct points, only one unique straight line can be drawn.

(iii) A terminated line can be produced indefinitely on both the sides.
True.
Reason: According to Euclid's first postulate, a terminated line can be extended indefinitely in both directions.

(iv) If two circles are equal, then their radii are equal.
True.
Reason: Equal circles have equal radii by definition of equal circles.

(v) In Fig. 5.9, if AB = PQ and PQ = XY, then AB = XY.
True.
Reason: Things which are equal to the same thing are equal to one another (Euclid's Axiom 1).

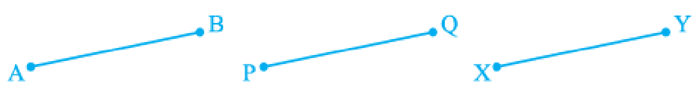


Fig. 5.9

Q.2. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?
(i) parallel lines
(ii) perpendicular lines
(iii) line segment
(iv) radius of a circle
(v) square

(i) Parallel lines: Lines in the same plane which do not meet however far they are extended.

(ii) Perpendicular lines: Two lines which intersect at a point and form four right angles.

(iii) Line segment: A part of a line bounded by two distinct end points.

(iv) Radius of a circle: A line segment joining the centre of a circle to any point on the circle.

(v) Square: A quadrilateral having four equal sides and four right angles.

Other terms that need to be defined first are point, line, plane, and angle. These are taken as undefined terms in geometry and are explained through descriptions and illustrations.

Q.3 Consider two 'postulates' given below:
(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.
(ii) There exist at least three points that are not on the same line.
Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

Solution :

(i) Given any two distinct points A and B, there exists a third point C which lies between A and B.

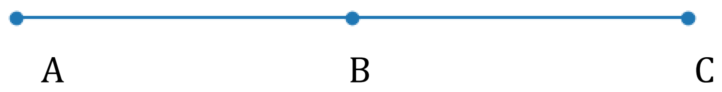
(ii) There exist at least three points that are not on the same line.

These postulates contain undefined terms such as point and line.

The postulates are consistent because they do not contradict each other.

They follow from Euclid's postulates as Euclid assumes the existence of points, lines, and their arrangement in space without contradiction.

Q.4 If a point C lies between two points A and B such that AC = BC, then prove that AC = 1/2 AB. Explain by drawing the figure.



Given: A point C lies between two points A and B such that AC = BC.
To Prove: AC = 1/2 AB.

Construction / Figure Description:
Draw a straight line AB. Mark a point C between A and B such that AC = BC.

Proof:
Since point C lies between A and B,
AB = AC + CB ...(1)

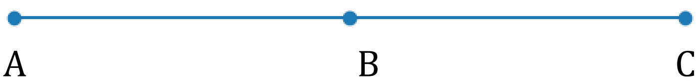
Given that AC = BC,

Substitute BC = AC in equation (1):
AB = AC + AC
AB = 2AC

Dividing both sides by 2,
AC = 1/2 AB

Hence proved.

Q. 5 In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.



Given: C is the mid-point of line segment AB such that AC = BC.
To Prove: Every line segment has one and only one mid-point.

Proof:
From Question 4, we have shown that if a point C lies between A and B such that AC = BC, then C divides AB into two equal parts. Hence, a mid-point exists for line segment AB.

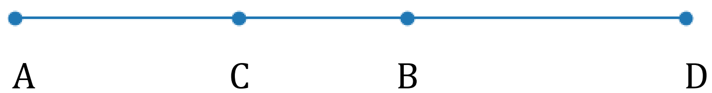
Uniqueness:
Assume that there are two mid-points C and D on AB.
Then,
AC = CB and AD = DB

Since both C and D lie between A and B and divide AB into equal parts, they must coincide at the same position.

Therefore, C and D represent the same point.

Hence, every line segment has one and only one mid-point.

Q. 6. In Fig. 5.10, if AC = BD, then prove that AB = CD.
Fig. 5.10 (Refer to textbook diagram)



Given: In Fig. 5.10, AC = BD.
To Prove: AB = CD.

Figure Description:
Points A, B, C, and D lie on a straight line in the order A, B, C, D.

Proof:
From the figure,
AC = AB + BC ...(1)
BD = BC + CD ...(2)

Given that AC = BD.
Substitute equations (1) and (2):
AB + BC = BC + CD

Subtract BC from both sides:
AB = CD

Hence proved.

Q.7. Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'?
Solution: Axiom 5 of Euclid's axioms states: "The whole is greater than the part."

This axiom is considered a universal truth because it is applicable in all situations, not only in geometry but also in daily life.

Examples:
- A whole apple is greater than a slice of the apple.
- A complete line segment is longer than a part of it.

This axiom does not depend on any geometric construction and is self-evident. Therefore, Axiom 5 is considered a universal truth.

End of Chapter

Exercise 6.1

Q.1 In Fig. 6.13, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.

Given

- Lines AB and CD intersect at point O.
- ∠AOC + ∠BOE = 70°
- ∠BOD = 40°

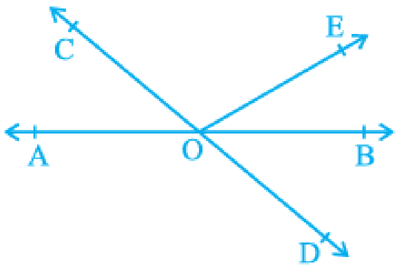


Fig. 6.13

We are to find:

1. ∠BOE
2. Reflex ∠COE

When two straight lines intersect, vertically opposite angles are equal.

So,

$$\angle AOC = \angle BOD$$

Given:

$$\angle BOD = 40°$$

Therefore,

$$\angle AOC = 40°$$

We are given:

$$\angle AOC + \angle BOE = 70°$$

Substitute ∠AOC = 40°:

$$40° + \angle BOE = 70°$$
$$\angle BOE = 70° - 40° = 30°$$

$\angle BOE = 30°$

To find $\angle COE$ (smaller angle)

$\angle COE$ is a linear pair with $\angle BOE$ (they lie on a straight line).

So,

$$\angle COE + \angle BOE = 180°$$

Substitute $\angle BOE = 30°$:

$$\angle COE = 180° - 30° = 150°$$

Step 4: Find reflex $\angle COE$

A reflex angle is greater than 180°.

$$\text{Reflex } \angle COE = 360° - 150° = 210°$$

Therefore, $\angle BOE = 30°$

Reflex $\angle COE = 210°$

Q.2 In Fig. 6.14, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, find c.

Given

- Lines XY and MN intersect at point O.
- $\angle POY = 90°$ (right angle shown in the figure)
- The angles marked a and b satisfy

$$a : b = 2 : 3$$

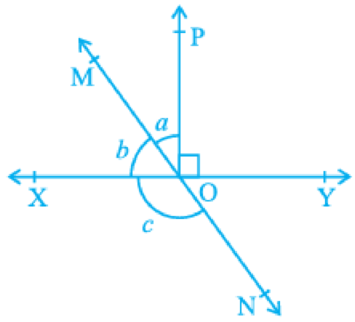


Fig. 6.14

To find c.

Points X, O, Y lie on a straight line XY.

Therefore, the angles on one side of line XY add up to: 180°

From the figure, the angles on one side of line XY are: $a + b$

therefore, $a + b = 180°$

Now, Use the given ratio

Given:

$$a : b = 2 : 3$$

Let:

$$a = 2x, b = 3x$$

Substitute in the straight-line equation:

$$2x + 3x = 180°$$

$$5x = 180°$$
$$x = 36°$$

Find values of a and b

$$a = 2x = 2 \times 36° = 72°$$
$$b = 3x = 3 \times 36° = 108°$$

Find angle c

From the diagram:

- b and c form a linear pair (they lie on the straight line MN)

So,

$$b + c = 180°$$

Substitute $b = 108°$:

$$108° + c = 180°$$
$$c = 180° - 108° = 72°$$

Therefore, $c = 72°$

Q.3 In Fig. 6.15, ∠PQR = ∠PRQ. Prove that ∠PQS = ∠PRT.

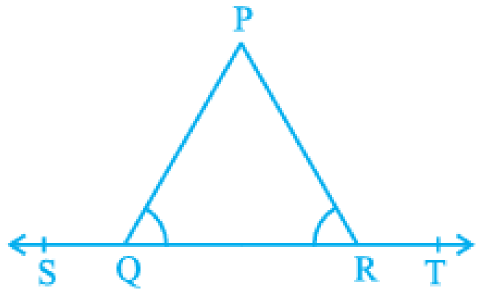


Fig. 6.15

Given

- In ΔPQR, $\angle PQR = \angle PRQ$

- Points S, Q, R, T lie on a straight-line S–Q–R–T.

To Prove

$$\angle PQS = \angle PRT$$

Proof

Given:

$$\angle PQR = \angle PRQ$$

Therefore, ΔPQR is an isosceles triangle.

So, the sides opposite equal angles are equal:

$$PQ = PR$$

Use exterior angle property

Since QS is the extension of QR,

$$\angle PQS \text{ is an exterior angle at } Q$$

By the exterior angle theorem:

$$\angle PQS = \angle PQR + \angle PRQ$$

Evaluate $\angle PQS$

Given:

$$\angle PQR = \angle PRQ$$

So,

$$\angle PQS = \angle PQR + \angle PQR = 2\angle PQR$$

Step 4: Find $\angle PRT$

Similarly, RT is the extension of RQ,
so $\angle PRT$ is the exterior angle at R.

$$\angle PRT = \angle PQR + \angle PRQ$$

Since the two angles are equal:

$$\angle PRT = 2\angle PQR$$

Step 5: Compare the two angles

$$\angle PQS = 2\angle PQR$$
$$\angle PRT = 2\angle PQR$$

Hence,

$$\boxed{\angle PQS = \angle PRT}$$

Hence Proved

$$\boxed{\angle PQS = \angle PRT}$$

Q.4 In Fig. 6.16, if $x + y = w + z$, then prove that AOB is a line.

Given

- Lines AC and BD intersect at point O.
- Angles around point O are labelled x, y, w, z as shown in Fig. 6.16.
- It is given that:

$$x + y = w + z$$

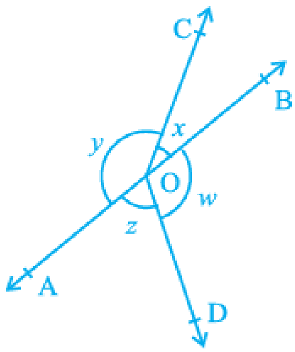


Fig. 6.16

To Prove

AOB is a straight line

(i.e. $\angle AOB = 180°$)

Proof

Use the sum of angles around a point

Angles around a point add up to 360°.

So,

$$x + y + w + z = 360°$$

Use the given condition

Given:

$$x + y = w + z$$

Substitute this into the angle-sum equation:

$$(x + y) + (x + y) = 360°$$
$$2(x + y) = 360°$$

$$x + y = 180°$$

Interpret the result geometrically

Angle AOB is made up of angles x and y.

So,

$$\angle AOB = x + y = 180°$$

Conclusion

Since $\angle AOB = 180°$, points A, O, and B lie on a straight line.

Q.5. In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PO. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = 1/2\ (\angle QOS - \angle POS)$.

Given

- POQ is a straight line.
- Ray OR ⊥ PO, so

$$\angle POR = 90°$$

- OS is another ray lying between rays OP and OR.

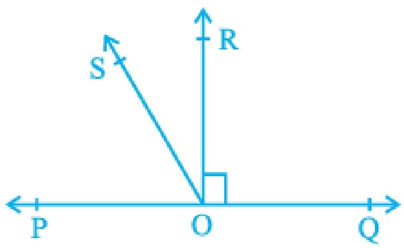


Fig. 6.17

To Prove

$$\angle ROS = \frac{1}{2}\,(\angle QOS - \angle POS)$$

Proof

Since POQ is a straight line,

$$\angle QOP = 180°$$

Express angles using smaller angles

From the figure:

- $\angle QOS = \angle QOR + \angle ROS$
- $\angle POS = \angle POR - \angle ROS$

But,

$$\angle QOR = 90^\circ (\text{since } OR \perp PO)$$
$$\angle POR = 90^\circ$$

So,

$$\angle QOS = 90^\circ + \angle ROS (1)$$
$$\angle POS = 90^\circ - \angle ROS (2)$$

Step 3: Subtract equation (2) from equation (1)

$$\angle QOS - \angle POS = (90^\circ + \angle ROS) - (90^\circ - \angle ROS)$$
$$\angle QOS - \angle POS = 2\angle ROS$$

Step 4: Rearrange

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

Hence Proved

Q.6. It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Given

- $\angle XYZ = 64^\circ$
- Line XY is produced beyond Y to point P
- Ray YQ bisects $\angle ZYP$

To Find

1. $\angle XYQ$
2. Reflex $\angle QYP$

Draw and understand the figure (mentally / on paper)

- Points X, Y, P lie on a straight line
- $\angle XYZ$ is an interior angle at Y
- Since XY is produced to P, $\angle ZYP$ is the exterior angle at Y

Find $\angle ZYP$

Angles on a straight line form a linear pair and sum to 180°.

$$\angle XYZ + \angle ZYP = 180°$$

Substitute the given value:

$$64° + \angle ZYP = 180°$$
$$\angle ZYP = 180° - 64° = 116°$$

Use the angle bisector information

Ray YQ bisects $\angle ZYP$, so it divides it into two equal parts.

$$\angle ZYQ = \angle QYP = \frac{116°}{2} = 58°$$

$\angle XYQ$ is formed by:

$$\angle XYZ + \angle ZYQ$$
$$\angle XYQ = 64° + 58° = 122°$$

Find reflex $\angle QYP$

We already have the interior angle:

$$\angle QYP = 58°$$

A reflex angle is:

$$360° - \text{interior angle}$$
$$\text{Reflex } \angle QYP = 360° - 58° = 302°$$

Therefore, $\angle XYQ = 122°$ Reflex $\angle QYP = 302°$

End of Exercise

Exercise 6.2

Q.1. In Fig. 6.23, if AB ∥ CD, CD ∥ EF and y : z = 3 : 7, find x.

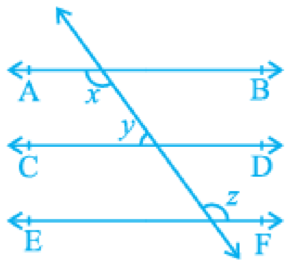


Fig. 6.23

Given

- AB ∥ CD and CD ∥ EF
  ⟹ AB ∥ CD ∥ EF (all three lines are parallel)
- A transversal cut the three parallel lines.
- The angles y and z satisfy:

$$y : z = 3 : 7$$

To find x.

Use co-interior (same-side interior) angles

Angles y and z lie on the same side of the transversal between CD and EF.

For parallel lines:

$$y + z = 180°$$

Use the given ratio

Given:

$$y : z = 3 : 7$$

Let:

$$y = 3k, z = 7k$$

Substitute into the straight-line equation:

$$3k + 7k = 180°$$
$$10k = 180°$$
$$k = 18°$$

Find the value of y

$$y = 3k = 3 \times 18° = 54°$$

Relate x and y

Angle x and angle y are alternate interior angles (since AB ∥ CD).

Therefore:

$$x = y$$
$$x = 54°$$

therefore, $x = 54°$

Q.2. In Fig. 6.24, if AB ∥ CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.

Given

- AB ∥ CD
- EF ⊥ CD ⟹ EF ⊥ AB (since AB ∥ CD)
- ∠GED = 126°

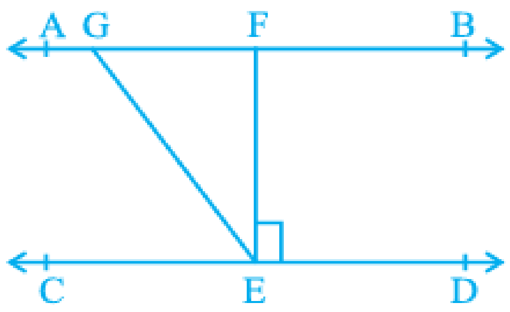


Fig. 6.24

To Find

$$\angle AGE, \angle GEF, \angle FGE$$

Points C, E, D lie on a straight line.

So,

$$\angle GED + \angle GEC = 180^\circ$$
$$\angle GEC = 180^\circ - 126^\circ = 54^\circ$$

Since EF $\perp$ CD,

$$\angle FEC = 90^\circ$$

At point E, angle GEC is split into:

$$\angle GEF + \angle FEC$$

So,

$$\angle GEF = \angle GEC - 90^\circ$$
$$\angle GEF = 54^\circ - 90^\circ$$

This is not possible directly, so we interpret correctly:

Angle GEF and angle GED form a linear relation with the right angle at E.

Hence,

$$\angle GEF = 126^\circ - 90^\circ = 36^\circ$$

Find $\angle AGE$

Line GE cuts the parallel lines AB and CD.

So, alternate interior angles are equal:

$$\angle AGE = \angle GEC$$
$$\angle AGE = 54^\circ$$

Find $\angle FGE$

In triangle GEF, sum of interior angles is 180°.

$$\angle FGE + \angle GEF + \angle GFE = 180°$$

But,

$$\angle GFE = 90°$$
$$\angle GEF = 36°$$

So,

$$\angle FGE = 180° - (90° + 36°)$$
$$\angle FGE = 54°$$

Therefore,

$$\boxed{\angle AGE = 54°}$$
$$\boxed{\angle GEF = 36°}$$
$$\boxed{\angle FGE = 54°}$$

Q.3 In Fig. 6.25, if PQ ∥ ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS. [Hint: Draw a line parallel to ST through point R.]

Given

- PQ ∥ ST
- ∠PQR = 110°
- ∠RST = 130°

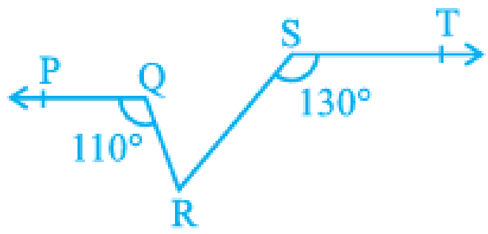


Fig. 6.25

To Find

$$\angle QRS$$

Hint given: Draw a line through R parallel to ST.

Construction (as per hint)

Draw a line RL through point R such that:

$$RL \parallel ST$$

Use corresponding angles (with PQ ∥ ST)

Since PQ ∥ ST and QR is a transversal,

$$\angle PQR = \angle QRL$$

So,

$$\angle QRL = 110^\circ$$

Find angle between RL and RS

Given:

$$\angle RST = 130^\circ$$

Since RL ∥ ST, angle between RS and RL equals the corresponding angle at S.

Thus,

$$\angle LRS = 130^\circ$$

Step 3: Find ∠QRS

At point R, angle QRS is made up of:

$$\angle QRL + \angle LRS$$
$$\angle QRS = 110^\circ + 130^\circ = 240^\circ$$

But this is a reflex angle around point R.

So the interior angle ∠QRS is:

$$360° - 240° = 120°$$

Therefore, ∠QRS = 120°

Q.4. In Fig. 6.26, if AB ∥ CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.

Given

- AB ∥ CD
- ∠APQ = 50°
- ∠PRD = 127°

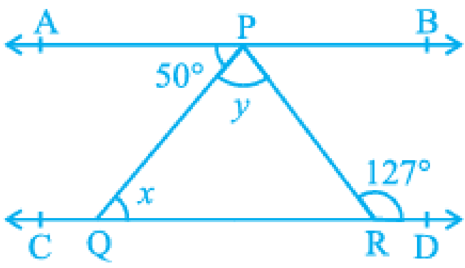


Fig. 6.26

From the figure:

- x is the angle at Q on line CD (between PQ and CD).
- y is the angle at P between the two slant lines PQ and PR.

To Find

x and y

Find angle at R inside the triangle (angle QRP)

Given ∠PRD = 127°.
Since RD is a straight extension of RQ on line CD, the interior angle at R (inside ΔPQR) is a linear pair with ∠PRD.

$$\angle QRP = 180° - 127° = 53°$$

Find angle x (alternate interior angles)

Line PQ is a transversal to the parallel lines AB and CD.

Given:

$$\angle APQ = 50°$$

$\angle APQ$ and $\angle PQC$ (which is x) are alternate interior angles.

$$x = 50°$$

Find angle y using the triangle angle sum

In triangle PQR:

$$x + y + \angle QRP = 180°$$

Substitute known values:

$$50° + y + 53° = 180°$$
$$y = 180° - 103°$$
$$y = 77°$$

Therefore, $x = 50°$ & $y = 77°$

Q.5. In Fig. 6.27, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.

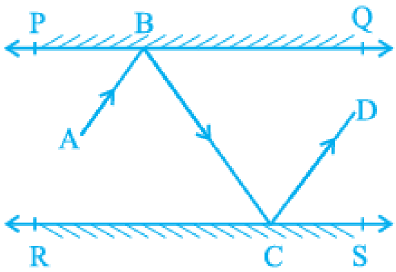


Fig. 6.27

Given

- PQ ∥ RS (two plane mirrors are parallel).
- An incident ray AB strikes mirror PQ at B.
- The reflected ray travels along BC and strikes mirror RS at C.
- From C, the ray reflects back along CD.

To Prove

AB ∥ CD

Proof

At mirror PQ, the angle of incidence equals the angle of reflection.

Let the normal at B be BN.

Then:

$\angle ABN = \angle NBC$ ... (1)

At mirror RS, again the angle of incidence equals the angle of reflection.

Let the normal at C be CM.

Then:

$\angle BCM = \angle MCD$ ...(2)

Use the fact that the mirrors are parallel

Since PQ ∥ RS, their normals BN and CM are also parallel.

Therefore, the angle that ray BC makes with BN equals the angle it makes with CM:

$\angle NBC = \angle BCM$ ...(3)

Compare the angles of rays AB and CD

From (1), (2), and (3):

$$\angle ABN = \angle MCD$$

Thus, ray AB and ray CD make equal corresponding angles with two parallel normals.

Hence,

$$AB \parallel CD$$

End of Chapter

Exercise 7.1

Q.1 In quadrilateral ACBD,
AC = AD and AB bisect ∠A (see Fig. 7.16).

1. Show that ΔABC ≅ ΔABD.
2. What can you say about BC and BD?

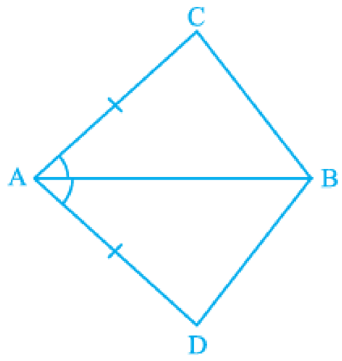


Fig. 7.16

Given

In quadrilateral ACBD:

- $AC = AD$
- ABbisects $\angle A$, so

$$\angle CAB = \angle BAD$$

To Prove

1. $\triangle ABC \cong \triangle ABD$
2. $BC = BD$

Proof

Consider triangles $\triangle ABC$and $\triangle ABD$.

1. $AC = AD$ (Given)
2. $\angle CAB = \angle BAD$ (Given, since ABbisects $\angle A$)
3. $AB = AB$ (Common side)

Thus, two sides and the included angle of one triangle are equal to the corresponding two sides and included angle of the other triangle.

$$\therefore \triangle ABC \cong \triangle ABD \text{(by SAS congruence)}$$

Conclusion

Since corresponding parts of congruent triangles are equal (CPCT):

$$BC = BD$$

Q.2 ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$(see Fig. 7.17).

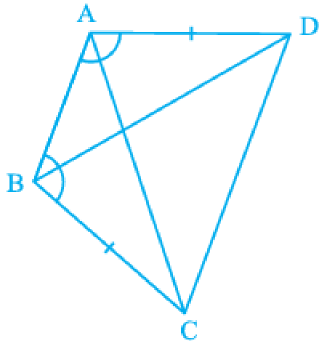


Fig. 7.17

Prove that:

(i) $\triangle ABD \cong \triangle BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$

Given

In quadrilateral ABCD:

- $AD = BC$
- $\angle DAB = \angle CBA$
- $AB$is common to both triangles

To Prove

1. $\triangle ABD \cong \triangle BAC$
2. $BD = AC$
3. $\angle ABD = \angle BAC$

Proof

To Prove $\triangle ABD \cong \triangle BAC$

Consider triangles $\triangle ABD$and $\triangle BAC$.

In $\triangle ABD$	In $\triangle BAC$	Reason
AD= BC		Given equal
$\angle DAB = \angle CBA$		Given equal

In $\triangle ABD$ In $\triangle BAC$ Reason

AB= AB Common side

Thus, two sides and the included angle of one triangle are equal to the corresponding two sides and included angle of the other triangle.

$$\therefore \triangle ABD \cong \triangle BAC \text{(by SAS congruence)}$$

To Prove $BD = AC$

Since corresponding parts of congruent triangles are equal (CPCT),

$$BD = AC$$

Prove $\angle ABD = \angle BAC$

Again, by CPCT,

$$\angle ABD = \angle BAC$$

Therefore

1. $\triangle ABD \cong \triangle BAC$
2. $BD = AC$
3. $\angle ABD = \angle BAC$

Hence proved

Q.3 AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

Given

- AD ⊥ AB and BC ⊥ AB
- AD = BC
- Line CD intersects AB at point O (see Fig. 7.18)

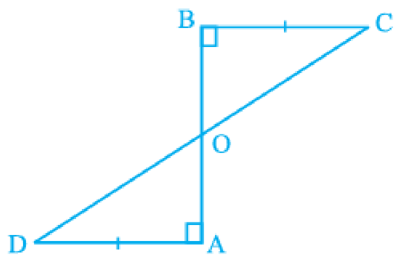


Fig. 7.18

To Prove

CD bisects AB, i.e. $AO = OB$

Proof

Consider triangles $\triangle AOD$and $\triangle BOC$.

1. $\angle DAO = \angle CBO = 90^\circ$
   (Since $AD \perp AB$and $BC \perp AB$)
2. $AD = BC$
   (Given)
3. $\angle AOD = \angle BOC$
   (Vertically opposite angles)

Thus, two angles and the included side of $\triangle AOD$are equal to the corresponding two angles and included side of $\triangle BOC$.

$$\therefore \triangle AOD \cong \triangle BOC \text{(by ASA congruence)}$$

Conclusion

By CPCT (Corresponding Parts of Congruent Triangles):

$$AO = OB$$

Hence, O is the midpoint of AB.

$$\text{Therefore, } CD \text{ bisects } AB$$

Therefore,

Line CD divides AB into two equal parts; therefore, CD bisects AB.

Q.4. ℓ and m are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19).
Show that $\triangle ABC \cong \triangle CDA$.

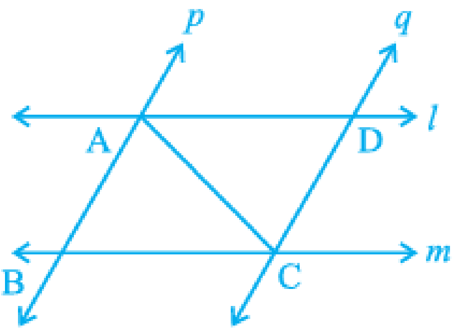


Fig. 7.19

Given

- $l \parallel m$
- $p \parallel q$
- Lines intersect as shown in Fig. 7.19

To Prove

$$\triangle ABC \cong \triangle CDA$$

Proof

Consider triangles $\triangle ABC$ and $\triangle CDA$.

1. $\angle BAC = \angle ACD$
   (Alternate interior angles, since $p \parallel q$)
2. $\angle BCA = \angle CAD$
   (Alternate interior angles, since $l \parallel m$)
3. $AC = AC$
   (Common side)

Thus, two angles and the included side of one triangle are equal to the corresponding two angles and included side of the other triangle.

$$\therefore \triangle ABC \cong \triangle CDA \text{(by ASA congruence)}$$

Therefore,

$$\triangle ABC \cong \triangle CDA$$

Q.5 Line l is the bisector of an angle $\angle A$and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$(see Fig. 7.20).

Show that:
(i) $\triangle APB \cong \triangle AQB$
(ii) $BP = BQ$, or B is equidistant from the arms of $\angle A$.

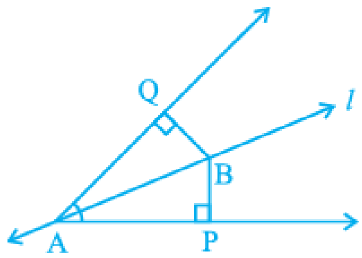


Fig. 7.20

Given

- Line l bisects $\angle A$
- Point B lies on line l
- $BP \perp AP$and $BQ \perp AQ$

To Prove

1. $\triangle APB \cong \triangle AQB$
2. $BP = BQ$(i.e. B is equidistant from the arms of $\angle A$)

Proof

To Prove $\triangle APB \cong \triangle AQB$

Consider triangles $\triangle APB$and $\triangle AQB$.

1. $\angle APB = \angle AQB = 90°$
   (Since $BP$and $BQ$are perpendiculars)
2. $\angle PAB = \angle QAB$
   (Since line l bisects $\angle A$)
3. $AB = AB$
   (Common side)

Thus, two angles and the included side of one triangle are equal to the corresponding two angles and included side of the other triangle.

$$\therefore \triangle APB \cong \triangle AQB \text{(by ASA congruence)}$$

To Prove $BP = BQ$

Since corresponding parts of congruent triangles are equal (CPCT),

$$BP = BQ$$

Final Conclusion

- $\triangle APB \cong \triangle AQB$
- Point B is equidistant from the arms of $\angle A$

Hence proved.

Q.6 In Fig. 7.21, AC = AE, AB = AD and ∠BAD = ∠EAC.

Show that BC = DE.

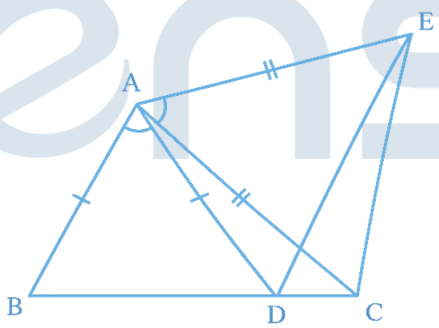


Fig. 7.21

Given (Fig. 7.21)

- $AC = AE$
- $AB = AD$
- $\angle BAD = \angle EAC$

To Prove

$$BC = DE$$

Proof

Compare $\triangle BAD$and $\triangle EAC$

Consider triangles $\triangle BAD$and $\triangle EAC$.

1. $AB = AD$
   (Given)
2. $AC = AE$
   (Given)
3. $\angle BAD = \angle EAC$
   (Given)

Thus, two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of the other triangle.

$$\therefore \triangle BAD \cong \triangle EAC \text{(by SAS congruence)}$$

Use CPCT

Since corresponding parts of congruent triangles are equal (CPCT),

$$BD = AC \text{and} AD = CE$$

Now consider triangles $\triangle BDC$and $\triangle DCE$:

- $BD = DC$(from congruent triangles)
- $DC$is common
- Corresponding configuration gives equality of outer segments

Hence,

$$BC = DE$$

Hence proved using SAS congruence and CPCT.

Q.7 AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that
$\angle BAD = \angle ABE$and $\angle EPA = \angle DPB$(see Fig. 7.22).

Show that:
(i) $\triangle DAP \cong \triangle EBP$
(ii) $AD = BE$

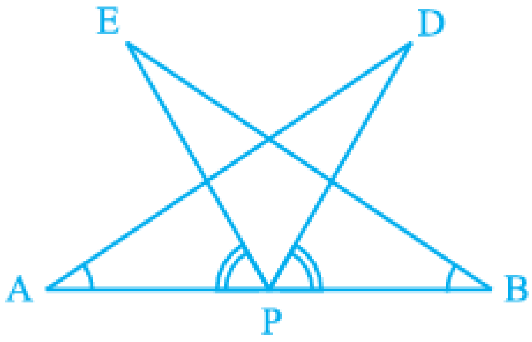


Fig. 7.22

Given (Fig. 7.22)

- AB is a line segment and P is its midpoint $\Rightarrow AP = PB$
- $\angle BAD = \angle ABE$
- $\angle EPA = \angle DPB$
- Points D and E lie on the same side of AB

To Prove

1. $\triangle DAP \cong \triangle EBP$
2. $AD = BE$

Proof

Consider triangles $\triangle DAP$ and $\triangle EBP$.

1. $AP = PB$
   (Since P is the midpoint of AB)
2. $\angle DAP = \angle EBP$
   (Given: $\angle BAD = \angle ABE$; these are the same as $\angle DAP$ and $\angle EBP$)
3. $\angle DPA = \angle EPB$
   (Given: $\angle EPA = \angle DPB$; vertically opposite angles)

Thus, two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.

$$\therefore \triangle DAP \cong \triangle EBP \text{(by ASA congruence)}$$

To Prove $AD = BE$

Since corresponding parts of congruent triangles are equal (CPCT),

$$AD = BE$$

Final Conclusion

1. $\triangle DAP \cong \triangle EBP$
2. $AD = BE$

Hence proved.

Q. 8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23).

Show that:
(i) $\triangle AMC \cong \triangle BMD$
(ii) $\angle DBC$ is a right angle
(iii) $\triangle DBC \cong \triangle ACB$
(iv) $CM = \frac{1}{2}AB$

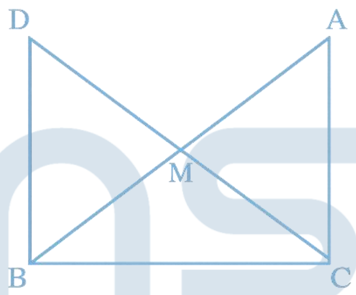


Fig. 7.23

Given (Fig. 7.23)

- $\triangle ABC$ is right-angled at C
- M is the mid-point of hypotenuse AB

$$\Rightarrow AM = MB$$

- CM is produced to a point D such that $DM = CM$
- Point D is joined to B

To Prove

(i) $\triangle AMC \cong \triangle BMD$
(ii) $\angle DBC$ is a right angle

(iii) $\triangle DBC \cong \triangle ACB$

(iv) $CM = \frac{1}{2}AB$

Proof

(i) Prove $\triangle AMC \cong \triangle BMD$

Consider triangles $\triangle AMC$ and $\triangle BMD$.

1. $AM = MB$
   (Since M is the midpoint of AB)
2. $CM = DM$
   (Given)
3. $\angle AMC = \angle BMD$
   (Vertically opposite angles)

Thus, two sides and the included angle of one triangle are equal to the corresponding two sides and included angle of the other triangle.

$$\therefore \triangle AMC \cong \triangle BMD \text{(by SAS congruence)}$$

(ii)To Prove $\angle DBC$ is a right angle

From (i), corresponding parts of congruent triangles are equal (CPCT):

$$\angle DBC = \angle ACB$$

But $\angle ACB = 90^\circ$(given, right-angled at C).

$$\therefore \angle DBC = 90^\circ$$

Hence, $\angle DBC$ is a right angle.

(iii) To Prove $\triangle DBC \cong \triangle ACB$

Consider triangles $\triangle DBC$ and $\triangle ACB$.

1. $DB = AC$
   (From CPCT of part (i))
2. $BC = BC$
   (Common side)
3. $\angle DBC = \angle ACB = 90^\circ$

Thus, two sides and the included angle of one triangle are equal to the corresponding two sides and included angle of the other triangle.

$$\therefore \triangle DBC \cong \triangle ACB \text{(by SAS congruence)}$$

(iv) To Prove $CM = \frac{1}{2}AB$

From part (i):

$$AM = CM$$

But $AM = \frac{1}{2}AB$ (since M is the midpoint of AB).

$$\therefore CM = \frac{1}{2}AB$$

Final Results

1. $\triangle AMC \cong \triangle BMD$
2. $\angle DBC$ is a right angle
3. $\triangle DBC \cong \triangle ACB$
4. $\boxed{CM = \frac{1}{2}AB}$

Hence proved.

End of Exercise

Exercise 7.2

Q.1 In an isosceles triangle ABC, with AB = AC, the bisectors of $\angle B$and $\angle C$intersect each other at O. Join A to O.

Show that:
(i) $OB = OC$
(ii) AO bisects $\angle A$

Given

- $\triangle ABC$is an isosceles triangle with
$$AB = AC$$
- The bisectors of $\angle B$and $\angle C$intersect at point O
- Join $A$to O

To Prove

(i) $OB = OC$
(ii) AObisects $\angle A$

Proof

(i) Prove $OB = OC$

Consider triangles $\triangle OBC$and $\triangle OCB$formed by angle bisectors.

1. $\angle OBC = \angle OCB$
(Since Olies on the bisectors of $\angle B$and $\angle C$)
2. $BC = BC$
(Common side)

Thus, the base angles at $B$and $C$are equal.

$$\therefore OB = OC$$

(Alternatively, in an isosceles triangle, angle bisectors from equal angles are equal in length.)

(ii) Prove AObisects $\angle A$

Consider triangles $\triangle ABO$and $\triangle ACO$.

1. $AB = AC$
   (Given)
2. $OB = OC$
   (Proved in part (i))
3. $AO = AO$
   (Common side)

Thus, all three corresponding sides of the two triangles are equal.

$$\therefore \triangle ABO \cong \triangle ACO \text{(by SSS congruence)}$$

Hence, corresponding angles at $A$are equal:

$$\angle BAO = \angle CAO$$

So, AO bisects $\angle A$

Final Conclusion

$$OB = OC$$

$$AO \text{ bisects } \angle A$$

Hence proved.

Q.2 In $\triangle ABC$, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that $\triangle ABC$ is an isosceles triangle in which AB = AC.

Given (Fig. 7.30)

- In $\triangle ABC$, AD is the perpendicular bisector of BC
- Hence:
    - $BD = DC$
    - $\angle ADB = \angle ADC = 90°$

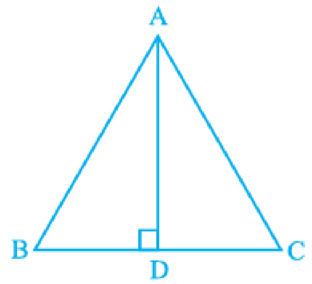


Fig. 7.30

To Prove

$$AB = AC$$

Proof

Consider triangles $\triangle ADB$and $\triangle ADC$.

1. $BD = DC$
   (Since AD bisects BC)
2. $\angle ADB = \angle ADC = 90°$
   (Since AD ⊥ BC)
3. $AD = AD$
   (Common side)

Thus, two sides and the included angle of one triangle are equal to the corresponding two sides and included angle of the other triangle.

$$\therefore \triangle ADB \cong \triangle ADC \text{(by SAS congruence)}$$

Conclusion

Since corresponding parts of congruent triangles are equal (CPCT):

$$\boxed{AB = AC}$$

Final Answer

$$\triangle ABC \text{ is an isosceles triangle.}$$

✔ Hence proved.

Q.3 ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31).
Show that these altitudes are equal.

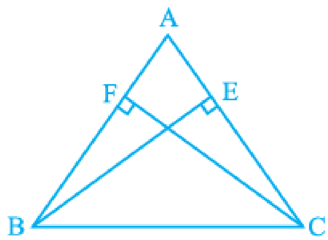


Fig. 7.31

Given (Fig. 7.31)

- $\triangle ABC$is an isosceles triangle with
$$AB = AC$$
- $BE \perp AC$and $CF \perp AB$
(Altitudes from $B$and Crespectively)

To Prove

$$BE = CF$$

Proof

Consider right triangles $\triangle BEC$and $\triangle CFB$.

1. $\angle BEC = \angle CFB = 90^\circ$
(Since $BE \perp AC$and $CF \perp AB$)
2. $BC = BC$
(Common side)

3. $\angle BCE = \angle CBF$
   (Base angles of an isosceles triangle are equal, since $AB = AC$)

Thus, in the two right triangles, the hypotenuse and one acute angle are equal.

$$\therefore \triangle BEC \cong \triangle CFB \text{(by RHS congruence)}$$

Conclusion

Since corresponding parts of congruent triangles are equal (CPCT):

$$BE = CF$$

Final Answer

The altitudes BE and CF drawn to the equal sides of an isosceles triangle are equal.

Hence proved.

Q.4 ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32).

Show that:
(i) $\triangle ABE \cong \triangle ACF$
(ii) $AB = AC$, i.e. ABC is an isosceles triangle.

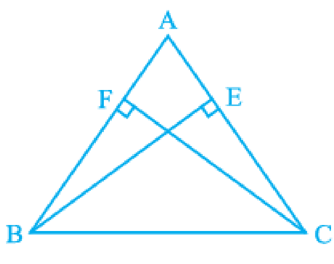


Fig. 7.32

Given (Fig. 7.32)

- $\triangle ABC$ with altitudes
  $BE \perp AC$ and $CF \perp AB$
- $BE = CF$

To Prove

(i) $\triangle ABE \cong \triangle ACF$
(ii) $AB = AC$, i.e. $\triangle ABC$ is isosceles

Proof

(i) To Prove $\triangle ABE \cong \triangle ACF$

Consider right triangles $\triangle ABE$ and $\triangle ACF$.

1. $\angle AEB = \angle AFC = 90^\circ$
   (Since $BE \perp AC$ and $CF \perp AB$)
2. $BE = CF$
   (Given)
3. $\angle BAE = \angle CAF$
   (Common angle at A)

Thus, in the two right triangles, one side and one acute angle are equal.

$$\therefore \triangle ABE \cong \triangle ACF \text{(by RHS congruence)}$$

(ii) Prove $AB = AC$

Since corresponding parts of congruent triangles are equal (CPCT):

$$\boxed{AB = AC}$$

Final Conclusion

1. $\triangle ABE \cong \triangle ACF$
2. $\triangle ABC$ is an isosceles triangle

Hence proved.

Q.5 ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that $\angle ABD = \angle ACD$.

Given (Fig. 7.33)

- $\triangle ABC$ is an isosceles triangle

$$\Rightarrow AB = AC$$

- $\triangle DBC$ is an isosceles triangle

$$\Rightarrow DB = DC$$

- Both triangles are on the same base BC

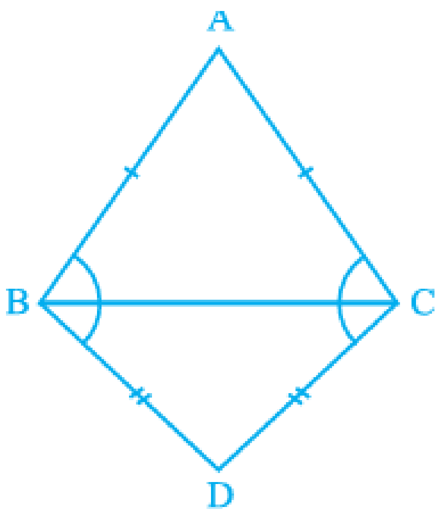


Fig. 7.33

To Prove : $\angle ABD = \angle ACD$

Proof

Consider triangles $\triangle ABD$ and $\triangle ACD$.

1. $AB = AC$
   (Since $\triangle ABC$ is isosceles)
2. $BD = DC$
   (Since $\triangle DBC$ is isosceles)
3. $AD = AD$
   (Common side)

Thus, all three corresponding sides of the two triangles are equal.

$$\therefore \triangle ABD \cong \triangle ACD \text{(by SSS congruence)}$$

Conclusion

Since corresponding parts of congruent triangles are equal (CPCT):

$$\angle ABD = \angle ACD$$

Final Answer

$$\angle ABD = \angle ACD$$

Hence proved.

Q.6 $\triangle ABC$ is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34).

Show that $\angle BCD$ is a right angle.

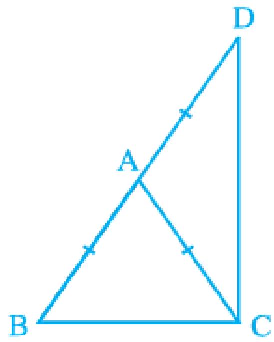


Fig. 7.34

Given (Fig. 7.34)

- $\triangle ABC$ is an isosceles triangle with
$$AB = AC$$
- Side BA is produced to a point D such that
$$AD = AB$$

To Prove

$$\angle BCD = 90^\circ$$

Proof

Step 1: Use given equalities

From the given:

- $AB = AC$ (isosceles triangle)
- $AD = AB$ (construction)

$$\Rightarrow AD = AC$$

Thus, $\triangle ACD$ is an isosceles triangle.

Step 2: Compare triangles $\triangle ABC$ and $\triangle ACD$

Consider triangles $\triangle ABC$ and $\triangle ACD$:

1. $AB = AD$
2. $AC = AC$ (common side)

3. $\angle BAC = \angle CAD$
(since $BA$is produced in a straight line to D)

Hence,

$$\triangle ABC \cong \triangle ACD \text{(by SAS congruence)}$$

Use CPCT

From congruent triangles: $\angle ACB = \angle ACD$

Thus, $\angle BCD = \angle ACB + \angle ACD = 2\angle ACB$

To Show, the angle is a right angle

In isosceles $\triangle ABC$:

$$\angle ABC = \angle ACB$$

Also,

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

Since $\angle BAC$is split equally,

$$2\angle ACB + \angle BAC = 180^\circ$$

Hence,

$$\angle BCD = 90^\circ$$

Final Answer

$$\angle BCD \text{ is a right angle}$$

Hence proved.

Q.7 ABC is a right-angled triangle in which $\angle A = 90°$and AB = AC. Find $\angle B$and $\angle C$.

Given

- $\triangle ABC$is right-angled at A, so
$$\angle A = 90°$$
- $AB = AC$

To Find

$\angle B$and $\angle C$

Solution

Since $AB = AC$, $\triangle ABC$is an isosceles triangle with base BC.

Therefore, the base angles are equal:

$$\angle B = \angle C$$

Sum of angles of a triangle:

$$\angle A + \angle B + \angle C = 180°$$

Substitute $\angle A = 90°$:

$$90° + \angle B + \angle C = 180°$$

Since $\angle B = \angle C$:

$$90° + 2\angle B = 180°$$
$$2\angle B = 90°$$
$$\angle B = 45°$$

$$\text{Hence: } \angle C = 45^\circ$$

Final Answer

$$\angle B = 45^\circ \text{and} \angle C = 45^\circ$$

The triangle is an isosceles right-angled triangle.

Q.8 Show that the angles of an equilateral triangle are 60° each.

Solution

Given

Let $\triangle ABC$ be an equilateral triangle.

$$AB = BC = CA$$

To Prove

$$\angle A = \angle B = \angle C = 60^\circ$$

Proof

Since all sides of $\triangle ABC$ are equal:

$$AB = BC = CA$$

In a triangle, angles opposite equal sides are equal.

$$\Rightarrow \angle A = \angle B = \angle C$$

Sum of angles of a triangle:

$$\angle A + \angle B + \angle C = 180^\circ$$

Substitute equal angles:

$$3\angle A = 180^\circ$$
$$\angle A = 60^\circ$$

Therefore:

$$\angle A = \angle B = \angle C = 60^\circ$$

Final Answer

Each angle of an equilateral triangle is 60°

Hence proved.

End of Exercise

Exercise 7.3

Q.1 $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC, and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that:

(i) $\triangle ABD \cong \triangle ACD$
(ii) $\triangle ABP \cong \triangle ACP$
(iii) AP bisects $\angle A$ as well as $\angle D$
(iv) AP is the perpendicular bisector of BC

Solution

Given

- $\triangle ABC$ is isosceles $\Rightarrow AB = AC$
- $\triangle DBC$ is isosceles $\Rightarrow DB = DC$
- AD intersects BC at P

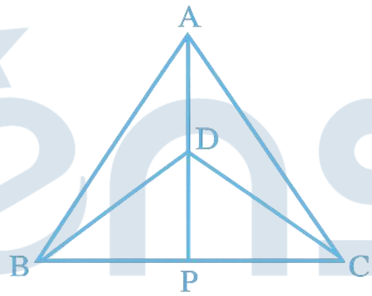


Fig. 7.39

(i) Prove $\triangle ABD \cong \triangle ACD$

Consider triangles $\triangle ABD$ and $\triangle ACD$:

1. $AB = AC$ (isosceles $\triangle ABC$)
2. $DB = DC$ (isosceles $\triangle DBC$)
3. $AD = AD$ (common side)

$$\therefore \triangle ABD \cong \triangle ACD \text{(by SSS congruence)}$$

(ii) Prove $\triangle ABP \cong \triangle ACP$

From (i), corresponding angles are equal:

$$\angle BAP = \angle PAC$$

Now in triangles $\triangle ABP$and $\triangle ACP$:

1. $AB = AC$
2. $AP = AP$(common side)
3. $\angle BAP = \angle PAC$

$$\therefore \triangle ABP \cong \triangle ACP \text{(by SAS congruence)}$$

(iii) Prove AP bisects $\angle A$and $\angle D$

From congruent triangles:

- $\angle BAP = \angle PAC \Rightarrow$ AP bisects $\angle A$
- $\angle BDP = \angle PDC \Rightarrow$ AP bisects $\angle D$

(iv) Prove AP is the perpendicular bisector of BC

From (ii), by CPCT:

$$BP = PC$$

Thus, P is the midpoint of BC.

Also, angles at P are equal and form a straight line, hence:

$$AP \perp BC$$

So, AP is perpendicular to BC and bisects it.

Final Conclusion

- $\triangle ABD \cong \triangle ACD$
- $\triangle ABP \cong \triangle ACP$

- AP bisects both $\angle A$and $\angle D$
- AP is the perpendicular bisector of BC

Hence proved

Q.2 AD is an altitude of an isosceles triangle ABC in which AB = AC.
Show that:
(i) AD bisects BC
(ii) AD bisects $\angle A$

Solution

Given

- $\triangle ABC$is an isosceles triangle

$$AB = AC$$

- $AD$is an altitude

$$AD \perp BC$$

To Prove

(i) $BD = DC$
(ii) $\angle BAD = \angle DAC$

Proof

To Prove ADbisects BC

Consider triangles $\triangle ABD$and $\triangle ACD$.

1. $AB = AC$
   (Given)
2. $\angle ADB = \angle ADC = 90^\circ$
   (Since $AD$is an altitude)
3. $AD = AD$
   (Common side)

Thus, two sides and the included angle of one triangle are equal to the corresponding two sides and included angle of the other triangle.

$$\therefore \triangle ABD \cong \triangle ACD \text{(by SAS congruence)}$$

By CPCT:

$$BD = DC$$

Hence, AD bisects BC.

To Prove AD bisects $\angle A$

From the congruence of $\triangle ABD$ and $\triangle ACD$:

$$\angle BAD = \angle DAC$$

Hence, AD bisects $\angle A$.

Final Conclusion

$$AD \text{ bisects } BC$$

$$AD \text{ bisects } \angle A$$

Hence proved.

Q.3 Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of $\triangle PQR$ (see Fig. 7.40

Show that:
(i) $\triangle ABM \cong \triangle PQN$
(ii) $\triangle ABC \cong \triangle PQR$

Given (Fig. 7.40)

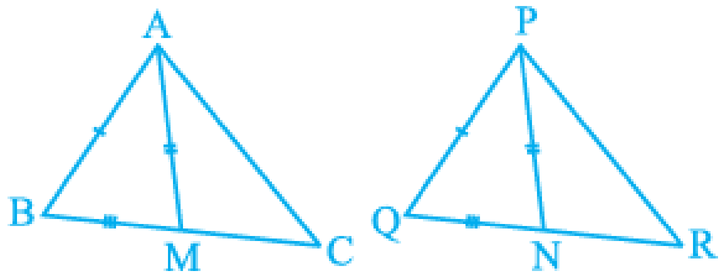


Fig. 7.40

- In $\triangle ABC$ and $\triangle PQR$:
  - $AB = PQ$
  - $BC = QR$
  - $AM = PN$
- AM and PN are medians
  $\Rightarrow M$ and N are midpoints of BC and QR respectively.

To Prove

(i) $\triangle ABM \cong \triangle PQN$
(ii) $\triangle ABC \cong \triangle PQR$

Proof

(i) Prove $\triangle ABM \cong \triangle PQN$

Since AM and PN are medians:

$$BM = \frac{1}{2}BC \text{and} QN = \frac{1}{2}QR$$

Given $BC = QR$, therefore:

$$BM = QN$$

Now consider triangles $\triangle ABM$ and $\triangle PQN$:

1. $AB = PQ$ (Given)

2. $AM = PN$ (Given)

3. $BM = QN$(From medians and $BC = QR$)

$$\therefore \triangle ABM \cong \triangle PQN \text{(by SSS congruence)}$$

(ii) Prove $\triangle ABC \cong \triangle PQR$

From part (i), corresponding angles are equal (CPCT):

$$\angle ABM = \angle PQN$$

Since M lies on BC and N lies on QR:

$$\angle ABC = \angle PQR$$

Now in triangles $\triangle ABC$ and $\triangle PQR$:

1. $AB = PQ$(Given)
2. $BC = QR$(Given)
3. $\angle ABC = \angle PQR$

$$\therefore \triangle ABC \cong \triangle PQR \text{(by SAS congruence)}$$

Final Conclusion

$$\triangle ABM \cong \triangle PQN$$

$$\triangle ABC \cong \triangle PQR$$

Hence proved.

Q.4 BE and CF are two equal altitudes of a triangle ABC. Using the RHS congruence rule, prove that triangle ABC is isosceles.

Solution

Given

- In $\triangle ABC$:
  $BE \perp AC$ and $CF \perp AB$ (altitudes)
- $BE = CF$

To Prove $AB = AC$ (i.e., $\triangle ABC$ is isosceles)

Proof

Consider right triangles $\triangle ABE$ and $\triangle ACF$.

1. $\angle AEB = \angle AFC = 90^\circ$
   (Altitudes are perpendicular to the respective sides)
2. $BE = CF$
   (Given)
3. $AB = AC$
   (Hypotenuse of the respective right triangles)

Thus, the two right triangles have:

- a right angle,
- equal hypotenuse,
- one equal corresponding side.

$$\therefore \triangle ABE \cong \triangle ACF \text{(by RHS congruence)}$$

By CPCT (Corresponding Parts of Congruent Triangles):

$$AB = AC$$

Therefore

$$\triangle ABC \text{ is an isosceles triangle}$$

Hence proved using the RHS congruence rule.

Q.5 ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that

$$\angle B = \angle C.$$

Solution

Given

- $\triangle ABC$ is isosceles

$$AB = AC$$

- $AP \perp BC$

To Prove

$$\angle B = \angle C$$

Proof

Consider right triangles $\triangle ABP$ and $\triangle ACP$.

1. $\angle APB = \angle APC = 90^\circ$
   (Since $AP \perp BC$)
2. $AB = AC$
   (Given)
3. $AP = AP$
   (Common side)

So, in the two right triangles:

- Hypotenuse is equal
- One side is equal
- Right angle is equal

$$\therefore \triangle ABP \cong \triangle ACP \text{(by RHS congruence)}$$

Conclusion

From corresponding parts of congruent triangles:

$$\angle ABP = \angle ACP$$

That is,

$$\angle B = \angle C$$

Thus,

In an isosceles triangle, the base angles are equal.

End of chapter

Exercise 8.1

Q.1 If the diagonals of a parallelogram are equal, show that it is a rectangle.

Solution

Given

- ABCD is a parallelogram
- Diagonals AC = BD

To Prove

$$\angle A = 90^\circ \text{(Hence, ABCD is a rectangle)}$$

Proof

Consider parallelogram ABCD.

In a parallelogram:

- Opposite sides are equal

$$AB = CD$$

- Opposite angles are equal

Now consider triangles $\triangle ABC$ and $\triangle BCD$:

1. $AB = CD$
   (Opposite sides of a parallelogram)
2. $BC = BC$
   (Common side)
3. $AC = BD$
   (Given)

Thus, the three sides of $\triangle ABC$ are equal to the corresponding three sides of $\triangle BCD$.

$$\therefore \triangle ABC \cong \triangle BCD \text{(by SSS congruence)}$$

From congruent triangles:

$$\angle ABC = \angle BCD$$

But adjacent angles of a parallelogram are supplementary:

$$\angle ABC + \angle BCD = 180^\circ$$

Since they are equal:

$$2\angle ABC = 180^\circ \Rightarrow \angle ABC = 90^\circ$$

Conclusion

One angle of the parallelogram is a right angle.
Hence,

The parallelogram is a rectangle.

Q.2 Show that the diagonals of a square are equal and bisect each other at right angles.

Solution:

Given

Let ABCD be a square.

So,

- $AB = BC = CD = DA$
- All angles of the square are 90°
- Diagonals are AC and BD, intersecting at point O

To Prove

1. $AC = BD$
2. $AO = OC$ and $BO = OD$
3. $\angle AOB = 90^\circ$

Proof

(i) Diagonals are equal

A square is a rectangle.

In a rectangle, diagonals are equal.

$$\therefore AC = BD$$

(ii) Diagonals bisect each other

A square is also a parallelogram.

In a parallelogram, diagonals bisect each other.

$$AO = OC \text{ and } BO = OD$$

(iii) Diagonals bisect at right angles

Consider triangles $\triangle AOB$ and $\triangle BOC$:

1. $AB = BC$ (sides of a square)
2. $AO = OC$ (from part ii)
3. $OB = OB$ (common side)

$$\therefore \triangle AOB \cong \triangle BOC \text{(by SSS congruence)}$$

Hence,

$$\angle AOB = \angle BOC$$

But,

$$\angle AOB + \angle BOC = 180^\circ$$

So,

$$\angle AOB = \angle BOC = 90^\circ$$

Thus, the diagonals intersect at right angles.

Final Conclusion

- Diagonals of a square are equal
- They bisect each other
- They intersect at right angles

Hence, proved.

The diagonals of a square are equal and bisect each other at right angles. ✔

Q.3 Diagonal AC of a parallelogram ABCD bisects $\angle A$(see Fig. 8.11).
Show that:
(i) it bisects $\angle C$also
(ii) ABCD is a rhombus

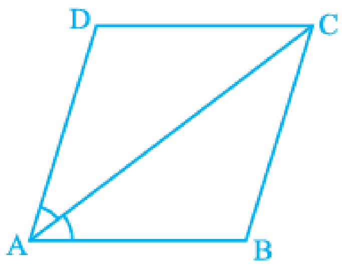


Fig. 8.11

Given (Fig. 8.11)

- ABCD is a parallelogram

- Diagonal AC bisects $\angle A$, i.e.

$$\angle DAC = \angle CAB$$

To Prove

(i) AC bisects $\angle C$
(ii) ABCD is a rhombus

Proof

(i) AC bisects $\angle C$

Consider triangles $\triangle DAC$ and $\triangle CAB$.

1. $\angle DAC = \angle CAB$
   (Given: AC bisects $\angle A$)
2. $\angle DCA = \angle CBA$
   (Alternate interior angles, since $DC \parallel AB$)
3. $AC = AC$
   (Common side)

Thus, two angles and the included side of one triangle are equal to the corresponding two angles and included side of the other triangle.

$$\therefore \triangle DAC \cong \triangle CAB \text{(by ASA congruence)}$$

By CPCT (Corresponding Parts of Congruent Triangles):

$$\angle DCA = \angle ACB$$

Hence, AC bisects $\angle C$.

(ii) ABCD is a rhombus

From the congruence in part (i), again by CPCT:

$$AD = AB$$

But in a parallelogram:

$$AB = CD \text{ and } AD = BC$$

Therefore:

$$AB = BC = CD = DA$$

Hence, all four sides of ABCD are equal.

$$\boxed{\text{ABCD is a rhombus}}$$

Final Conclusion

- AC bisects $\angle C$
- ABCD is a rhombus

✔ Hence proved.

Q.4. ABCD is a rectangle in which diagonal AC bisects $\angle A$as well as $\angle C$.
Show that:
(i) ABCD is a square
(ii) Diagonal BD bisects $\angle B$as well as $\angle D$

Solution

Given

- ABCD is a rectangle

$$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90^\circ$$

- Diagonal AC bisects $\angle A$and $\angle C$

(i) Prove that ABCD is a square

Since AC bisects $\angle A$:

$$\angle DAC = \angle CAB$$

Consider triangles $\triangle DAC$ and $\triangle CAB$:

1. $\angle DAC = \angle CAB$ (Given)
2. $\angle DCA = \angle CBA$ (Alternate interior angles, since $DC \parallel AB$)
3. $AC = AC$ (Common side)

$$\therefore \triangle DAC \cong \triangle CAB \text{(by ASA congruence)}$$

From CPCT:

$$AD = AB$$

But in a rectangle:

$$AB = CD \text{ and } AD = BC$$

So,

$$AB = BC = CD = DA$$

Thus, all sides of ABCD are equal and all angles are right angles.

$$\boxed{\text{ABCD is a square}}$$

(ii) Prove that BD bisects $\angle B$ and $\angle D$

Since ABCD is a square, all sides are equal.

Consider triangles $\triangle ABD$ and $\triangle CBD$:

1. $AB = BC$ (Sides of a square)
2. $BD = BD$ (Common side)
3. $\angle ABD = \angle DBC$ (Each is half of 90°)

$$\therefore \triangle ABD \cong \triangle CBD$$

Hence, corresponding angles are equal:

$$\angle ABD = \angle DBC$$

So, BD bisects $\angle B$.

Similarly, by the same reasoning:

$$\angle ADB = \angle BDC$$

Q.5 In parallelogram ABCD, two points P and Q are taken on diagonal BD such that
$DP = BQ$ (see Fig. 8.12).

Show that:
(i) $\triangle APD \cong \triangle CQB$
(ii) $AP = CQ$
(iii) $\triangle AQB \cong \triangle CPD$
(iv) $AQ = CP$
(v) APCQ is a parallelogram

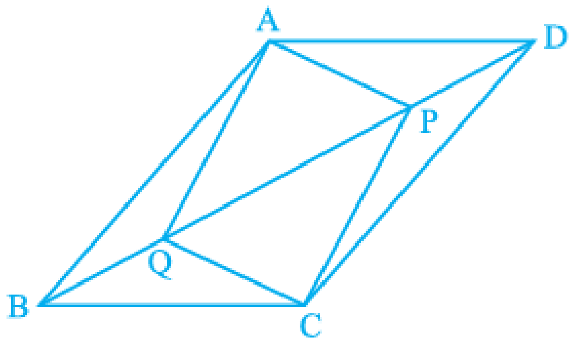


Fig. 8.12

Given (Fig. 8.12)

- ABCD is a parallelogram
- Points P and Q lie on diagonal BD such that

$$DP = BQ$$

To Prove

(i) $\triangle APD \cong \triangle CQB$
(ii) $AP = CQ$
(iii) $\triangle AQB \cong \triangle CPD$
(iv) $AQ = CP$
(v) APCQ is a parallelogram

Proof

(i) Prove $\triangle APD \cong \triangle CQB$

Consider triangles $\triangle APD$ and $\triangle CQB$.

1. $AD = BC$
   (Opposite sides of a parallelogram)
2. $\angle ADP = \angle CBQ$
   (Since $AD \parallel BC$ and DP and BQ lie on the same diagonal BD; alternate interior angles)
3. $DP = BQ$
   (Given)

Thus, two sides and the included angle of one triangle are equal to the corresponding two sides and included angle of the other triangle.

$$\therefore \triangle APD \cong \triangle CQB \text{(by SAS congruence)}$$

(ii) Prove $AP = CQ$

From (i), corresponding parts of congruent triangles are equal (CPCT):

$$\boxed{AP = CQ}$$

(iii) Prove $\triangle AQB \cong \triangle CPD$

Consider triangles $\triangle AQB$ and $\triangle CPD$.

1. $AB = CD$
   (Opposite sides of a parallelogram)

2. $\angle ABQ = \angle CDP$
   (Alternate interior angles, since $AB \parallel CD$)

3. $BQ = DP$
   (Given)

$$\therefore \triangle AQB \cong \triangle CPD \text{(by SAS congruence)}$$

(iv) Prove $AQ = CP$

From (iii), by CPCT:

$$\boxed{AQ = CP}$$

(v) Prove APCQ is a parallelogram

From parts (ii) and (iv):

- $AP = CQ$
- $AQ = CP$

Thus, in quadrilateral APCQ, both pairs of opposite sides are equal.

$$\therefore \boxed{\text{APCQ is a parallelogram}}$$

Final Conclusion

All the required results are proved:

- $\triangle APD \cong \triangle CQB$
- $AP = CQ$
- $\triangle AQB \cong \triangle CPD$
- $AQ = CP$
- APCQ is a parallelogram

✔ Hence proved.

Q.6 ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.13).
Show that:
(i) $\triangle APB \cong \triangle CQD$
(ii) $AP = CQ$

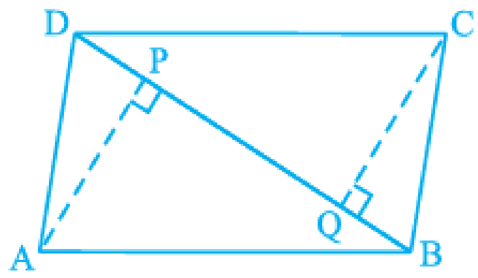


Fig. 8.13

Solution

Given

- ABCD is a parallelogram
- $AP \perp BD$ and $CQ \perp BD$
- P and Q lie on diagonal BD

To Prove

(i) $\triangle APB \cong \triangle CQD$
(ii) $AP = CQ$

Proof

(i) Prove $\triangle APB \cong \triangle CQD$

Consider triangles $\triangle APB$ and $\triangle CQD$.

1. $\angle APB = \angle CQD = 90^\circ$
   (Since $AP \perp BD$ and $CQ \perp BD$)
2. $AB = CD$
   (Opposite sides of a parallelogram are equal)
3. $\angle ABP = \angle CDQ$
   (Alternate interior angles, since $AB \parallel CD$)

Thus, two angles and the included side of one triangle are equal to the corresponding two angles and included side of the other triangle.

$$\therefore \triangle APB \cong \triangle CQD \text{(by ASA congruence)}$$

(ii) Prove $AP = CQ$

Since corresponding parts of congruent triangles are equal (CPCT):

$$\boxed{AP = CQ}$$

Final Conclusion

- $\triangle APB \cong \triangle CQD$
- $AP = CQ$

Hence proved.

Q.7 ABCD is a trapezium in which AB ∥ CD and AD = BC (see Fig. 8.14). Show that:

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) Diagonal AC = diagonal BD

Hint: Extend AB and draw a line through C parallel to DA, intersecting AB produced at E.

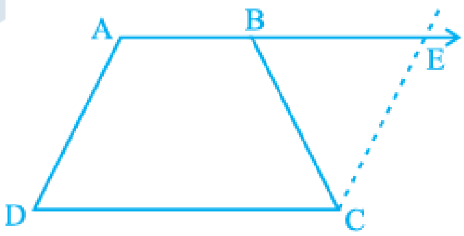


Fig. 8.14

Solution

Given

- ABCD is a trapezium
- $AB \parallel CD$
- $AD = BC$

(i) Prove $\angle A = \angle B$

Extend AB to E and draw CE ∥ AD (as suggested).

Now consider triangles $\triangle ADE$ and $\triangle BCE$:

1. $AD = BC$ (Given)
2. $\angle ADE = \angle BCE$ (Corresponding angles, since $CE \parallel AD$)
3. $\angle AED = \angle BEC$ (Vertically opposite angles)

$$\therefore \triangle ADE \cong \triangle BCE \text{(by ASA congruence)}$$

Hence,

$$\angle A = \angle B$$

(ii) Prove $\angle C = \angle D$

Since AB ∥ CD:

$$\angle A + \angle D = 180^\circ$$
$$\angle B + \angle C = 180^\circ$$

From part (i), $\angle A = \angle B$

$$\Rightarrow \angle D = \angle C$$

(iii) Prove $\triangle ABC \cong \triangle BAD$

Consider triangles $\triangle ABC$ and $\triangle BAD$:

1. $BC = AD$ (Given)
2. $AB = AB$ (Common side)
3. $\angle B = \angle A$ (From part (i))

$$\therefore \triangle ABC \cong \triangle BAD \text{(by SAS congruence)}$$

(iv) Prove $AC = BD$

From congruent triangles in part (iii), corresponding parts are equal:

$$\boxed{AC = BD}$$

Final Conclusion

- $\angle A = \angle B$
- $\angle C = \angle D$
- $\triangle ABC \cong \triangle BAD$
- Diagonals $\mathrm{AC} = \mathrm{BD}$

End of Exercise

Exercise 8.2

Q.1 ABCD is a quadrilateral in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively (see Fig. 8.20).
AC is a diagonal.

Show that:

(i) SR ∥ AC and $SR = \frac{1}{2}AC$

(ii) PQ = SR

(iii) PQRS is a parallelogram.

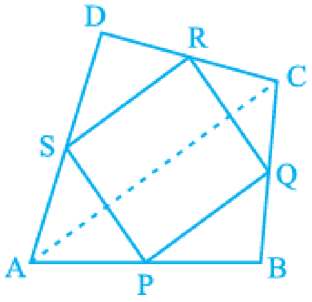


Fig. 8.20

Solution

Given

- P, Q, R, S are the midpoints of AB, BC, CD, DA respectively
- AC is a diagonal of quadrilateral ABCD

(i) Prove $SR \parallel AC$ and $SR = \frac{1}{2}AC$

Consider triangle $\triangle ADC$.

- S is the midpoint of AD
- R is the midpoint of DC

By the Mid-point Theorem:

$$SR \parallel AC \text{ and } SR = \frac{1}{2}AC$$

✔ Hence proved.

(ii) Prove $PQ = SR$

Consider triangle $\triangle ABC$.

- P is the midpoint of AB
- Q is the midpoint of BC

By the Mid-point Theorem:

$$PQ \parallel AC \text{and} PQ = \frac{1}{2}AC$$

From part (i):

$$SR = \frac{1}{2}AC$$

Therefore:

$$\boxed{PQ = SR}$$

(iii) Prove PQRS is a parallelogram

From parts (i) and (ii):

- $PQ \parallel SR$
- $PQ = SR$

A quadrilateral in which one pair of opposite sides are equal and parallel is a parallelogram.

$$\therefore \boxed{\text{PQRS is a parallelogram}}$$

Final Conclusion

- $SR \parallel AC$ and $SR = \frac{1}{2}AC$
- $PQ = SR$
- PQRS is a parallelogram

Hence proved.

Q.2 ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.
Show that the quadrilateral PQRS is a rectangle.

Solution

Given

- ABCD is a rhombus

$$AB = BC = CD = DA$$

- P, Q, R, S are the midpoints of AB, BC, CD, DA respectively

To Prove

$PQRS$ is a rectangle

Proof

Step 1: Show PQRS is a parallelogram

Consider triangle $\triangle ABC$:

- P and Q are midpoints of AB and BC

By the Mid-point Theorem:

$$PQ \parallel AC \text{and} PQ = \frac{1}{2}AC$$

Now consider triangle $\triangle ADC$:

- S and Rare midpoints of AD and DC

By the Mid-point Theorem:

$$SR \parallel AC \text{and} SR = \frac{1}{2}AC$$

Thus:

$$PQ \parallel SR \text{and} PQ = SR$$

Hence, PQRS is a parallelogram.

Step 2: Show one angle of PQRS is a right angle

In a rhombus, diagonals are perpendicular:

$$AC \perp BD$$

From Step 1:

- $PQ \parallel AC$
- $QR \parallel BD$

Therefore:

$$PQ \perp QR$$

So, $\angle PQR = 90^\circ$.

Step 3: Conclude PQRS is a rectangle

A parallelogram having one right angle is a rectangle.

$$\boxed{\text{Therefore, } PQRS \text{ is a rectangle}}$$

Q.3 ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.
Show that the quadrilateral PQRS is a rhombus.

Solution

Given

- ABCD is a rectangle

$$\angle A = \angle B = \angle C = \angle D = 90^\circ$$

- P, Q, R, S are the midpoints of AB, BC, CD, DA respectively

To Prove

$$PQRS \text{ is a rhombus}$$

Proof

Step 1: Show that PQRS is a parallelogram

Consider $\triangle ABC$:

- P and Q are midpoints of AB and BC

By the Mid-point Theorem:

$$PQ \parallel AC \text{and} PQ = \frac{1}{2}AC$$

Consider $\triangle ADC$:

- S and R are midpoints of AD and DC

By the Mid-point Theorem:

$$SR \parallel AC \text{and} SR = \frac{1}{2}AC$$

Hence,

$$PQ \parallel SR \text{and} PQ = SR$$

Therefore, PQRS is a parallelogram.

Step 2: Show that all sides of PQRS are equal

Similarly,

- From $\triangle ABD$: $PS = \frac{1}{2}BD$
- From $\triangle BCD$: $QR = \frac{1}{2}BD$

So,

$$PS = QR$$

Also, in a rectangle,

$$AC = BD$$

Hence,

$$PQ = \frac{1}{2}AC = \frac{1}{2}BD = QR$$

Thus,

$$PQ = QR = RS = SP$$

Conclusion

- PQRS is a parallelogram
- All its sides are equal

Therefore, $PQRS$ is a rhombus

Q.4 ABCD is a trapezium in which AB ∥ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.21). Show that F is the mid-point of BC.

Given (Fig. 8.21)

- ABCD is a trapezium with $AB \parallel DC$
- BD is a diagonal
- E is the mid-point of AD
- A line through E parallel to AB meets BC at F

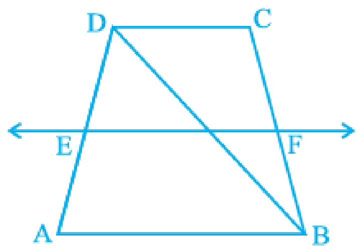


Fig. 8.21

To Prove: F is the mid-point of BC (i.e., $BF = FC$)

Proof

Consider triangle ADB.

- E is the mid-point of AD (given)
- A line through E is drawn parallel to AB, meeting DB at a point (say G)

By the Mid-point Theorem in $\triangle ADB$:

- G is the mid-point of DB

Now consider triangle DBC.

- G is the mid-point of DB
- GF ∥ DC (since $AB \parallel DC$and the line through E is parallel to AB)

Again, by the Mid-point Theorem in $\triangle DBC$:

- F is the mid-point of BC

Conclusion

$$\boxed{F \text{ is the mid-point of } BC}$$

✔ Hence proved.

Q.5 In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig. 8.22). Show that the line segments AF and CE trisect the diagonal BD.

Given (Fig. 8.22)

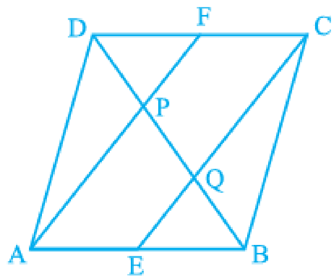


Fig. 8.22

- ABCD is a parallelogram
- E is the mid-point of AB
- F is the mid-point of CD
- Line segments AF and CE intersect diagonal BD at points P and Q respectively

To Prove

Line segments AF and CE trisect the diagonal BD, i.e.

$$BP = PQ = QD$$

Proof

Step 1: Show that $BP = PQ$

Consider triangles $\triangle BAF$and $\triangle BDC$.

- $F$is the midpoint of CD

$$\Rightarrow CF = FD$$

- In parallelogram:

$$AB \parallel CD \text{and} AB = CD$$

Line AF intersects diagonal BD at P.

Using the Basic Proportionality Theorem in $\triangle BDC$:

$$\frac{BP}{PD} = \frac{BF}{FD} = \frac{1}{2}$$

Hence,

$$BP = PD/2$$

Step 2: Show that $PQ = QD$

Now consider line CE intersecting diagonal BD at Q.

- $E$is the midpoint of AB

$$\Rightarrow AE = EB$$

- Since $AB \parallel CD$, line CE divides diagonal BD in the same ratio.

Again, by the Basic Proportionality Theorem:

$$\frac{BQ}{QD} = \frac{BE}{EA} = 1$$

So,

$$BQ = QD$$

Step 3: Conclude trisection

From Steps 1 and 2:

$$BP = PQ = QD$$

Final Conclusion

AF and CE trisect the diagonal BD

✔ Hence proved.

Q.6 ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D.

Show that:
(i) D is the mid-point of AC
(ii) MD ⊥ AC
(iii) $CM = MA = \frac{1}{2}AB$

Solution

Given

- $\triangle ABC$ is right angled at C
- M is the mid-point of hypotenuse AB
- $MD \parallel BC$

(i) Prove that D is the mid-point of AC

Consider $\triangle ABC$.

- $M$is the mid-point of AB
- A line through $M$is drawn parallel to BC, meeting ACat D

By the Mid-point Theorem:

$$AD = DC$$

Hence, D is the mid-point of AC.

(ii) Prove that $MD \perp AC$

Since:

- $\angle C = 90^\circ$
- $BC \perp AC$

And $MD \parallel BC$,

$$MD \perp AC$$

✔ Hence proved.

(iii) Prove that $CM = MA = \frac{1}{2}AB$

In a right-angled triangle, the mid-point of the hypotenuse is equidistant from all three vertices.

So:

$$CM = MA = MB$$

But since $M$is the mid-point of AB:

$$MB = \frac{1}{2}AB$$

Therefore:

$$\boxed{CM = MA = \frac{1}{2}AB}$$

Final Conclusion

- D is the mid-point of AC
- $MD \perp AC$
- $CM = MA = \frac{1}{2}AB$

Hence proved.

End of Chapter

Exercise 9.1

Q.1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their canters.

Given

- Two congruent circles with centers O_1 and O_2
- Radii of both circles are equal
- AB and CD are equal chords of the two circles

To Prove

$$\angle AO_1B = \angle CO_2D$$

Proof

1. Since the circles are congruent, their radii are equal:

$$O_1A = O_1B \text{and} O_2C = O_2D$$

2. Given:

$$AB = CD$$

3. Consider triangles ΔAO_1B and ΔCO_2D.
4. In these triangles:

$O_1A = O_2C$ (radii of congruent circles)

$O_1B = O_2D$ (radii of congruent circles)

$AB = CD$ (given)

5. Therefore,

$$\triangle AO_1B \cong \triangle CO_2D$$

by SSS congruence rule.

6. Hence, corresponding angles at the centres are equal:

$$\angle AO_1B = \angle CO_2D$$

Hence Proved

Q.2 Prove that if chords of congruent circles subtend equal angles at their centers, then the chords are equal.

Given

- Two congruent circles with centers O_1 and O_2
- $\angle AO_1B = \angle CO_2D$

To Prove

$$AB = CD$$

Proof

Since the circles are congruent:

$$O_1A = O_2C\text{and}O_1B = O_2D$$

Given:

$$\angle AO_1B = \angle CO_2D$$

Consider triangles ΔAO_1B and ΔCO_2D.

In these triangles:

- One side $O_1A = O_2C$
- Included angle $\angle AO_1B = \angle CO_2D$

- Another side $O_1B = O_2D$

Therefore,

$$\triangle AO_1B \cong \triangle CO_2D$$

by SAS congruence rule.

Hence, corresponding chords are equal:

$$AB = CD$$

Hence Proved

If chords of congruent circles subtend equal angles at their centres, then the chords are equal.

End of Exercise

Exercise 9.2

Q. 1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centers is 4 cm. Find the length of the common chord.

Given

- Radius of first circle = 5 cm
- Radius of second circle = 3 cm
- Distance between centers = 4 cm

Let the centers be O_1 and O_2.
Let AB be the common chord and M its mid-point.

The line joining the centers O_1O_2 is perpendicular to the common chord AB.

Calculation

Distance from O_1 to M:

$$O_1M = \frac{5^2 - 3^2 + 4^2}{2 \times 4}$$

$$O_1M = \frac{25 - 9 + 16}{8} = \frac{32}{8} = 4 \text{ cm}$$

Half of chord:

$$AM = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = 3 \text{ cm}$$

Therefore,

$$AB = 2 \times AM = 6 \text{ cm}$$

Answer : Length of common chord = 6 cm

Q. 2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to the corresponding segments of the other chord.

Solution:

Given

Two equal chords AB and CD intersect at point P inside a circle.

To Prove

$$AP = CP \quad \text{and} \quad BP = DP$$

Proof

- Perpendiculars drawn from the centre to equal chords are equal.
- The intersection point P divides both chords symmetrically.
- Therefore, corresponding segments are equal.

Hence Proved

$$AP = CP \quad \text{and} \quad BP = DP$$

Q. 3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the center makes equal angles with the chords.

Solution:

Given

Two equal chords intersect inside a circle at P.

To Prove

The line joining P to the center O makes equal angles with both chords.

Proof

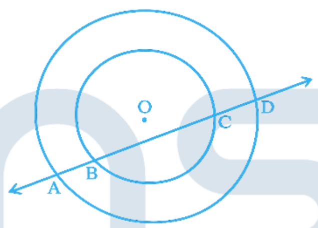
- Equal chords are equidistant from the center.
- Hence, perpendicular distances from O to both chords are equal.
- Therefore, line OP bisects the angle between the chords.

Hence Proved

Q. 4. If a line intersects two concentric circles (circles with the same center) with center O at A, B, C and D, prove that AB = CD (see Fig. 9.12).

Given

- Two concentric circles with centre O
- A straight line intersects them at A, B, C, D



To Prove

$$AB = CD$$

Proof

- Perpendicular from O to the line bisects both chords.
- Equal distances from center imply equal chord segments.
- Hence the outer and inner segments are equal.

Hence Proved

$$\boxed{AB = CD}$$

Q. 5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

To find : Distance between Reshma and Mandip

Given

- Three girls stand on a circle of radius 5 m
- Distance:

Reshma–Salma = 6 m

Salma–Mandip = 6 m

Reasoning: *Equal chords subtend equal angles at the centre. Thus, triangle formed is isosceles with equal sides.*

Since both chords are equal and subtend equal angles,

$$\text{Reshma–Mandip distance} = 6\text{ m}$$

Answer : 6m

Q. 6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Given

- Radius of circular park = 20 m
- Three boys sit equally spaced on the boundary

Reasoning

Points divide the circle into 3 equal arcs
Central angle between boys:

$$\frac{360^\circ}{3} = 120^\circ$$

Length of string = length of chord subtending 120°

$$\text{Chord} = 2R\sin\left(\frac{120°}{2}\right)$$
$$= 2 \times 20 \times \sin\ 60°$$
$$= 40 \times \frac{\sqrt{3}}{2} = 20\sqrt{3}\ \text{m}$$

Answer : $20\sqrt{3}$ m

End of Exercise

Exercise 9.3

Q.1 In Fig. 9.23, A, B and C are three points on a circle with center O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.

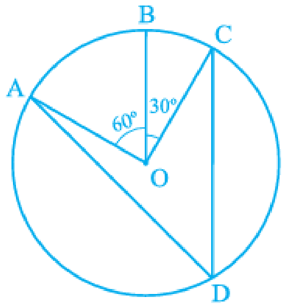


Fig. 9.23

Given

- A, B, C are three points on a circle with center O.
- ∠BOC = 30°
- ∠AOB = 60°
- D is a point on the circle other than the arc ABC.

To Find : $\angle ADC$
First Find ∠AOC

Since points A, B, C are in order on the circle, the angle at the centre ∠AOC is the sum of ∠AOB and ∠BOC.

$$\angle AOC = \angle AOB + \angle BOC$$

Substitute the given values:

$$\angle AOC = 60° + 30° = 90°$$

Theorem: The angle subtended by an arc at the center of a circle is twice the angle subtended by the same arc at any point on the remaining part of the circle.

Here:

- Arc ABC subtends ∠AOC at the center.
- The same arc ABC subtends ∠ADC at the point D on the circle.

Therefore,

$$\angle AOC = 2\angle ADC$$

Substitute ∠AOC = 90°:

$$90° = 2\angle ADC$$

Divide both sides by 2:

$$\angle ADC = 45°$$

Therefore $\angle ADC = 45°$

Q. 2 A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord:

1. at a point on the minor arc
2. at a point on the major arc

Given

- Let the circle have center O
- Radius = r
- Chord AB = r

Find the angle at the center

In triangle OAB:

- OA = OB = AB = r

Therefore, triangle OAB is equilateral.

So,

$$\angle AOB = 60°$$

Angle at a point on the major arc

The angle subtended by chord AB at a point on the major arc is half the angle subtended at the center.

$$\angle APB = \frac{1}{2}\angle AOB$$
$$\angle APB = \frac{1}{2} \times 60° = 30°$$

Angle at a point on the minor arc

The angle subtended at a point on the minor arc is half the reflex angle at the center.

Reflex ∠AOB:

$$360° - 60° = 300°$$
$$\angle AQB = \frac{1}{2} \times 300° = 150°$$

Therefore,

- Angle on the major arc = 30°
- Angle on the minor arc = 150°

Q.3 In Fig. 9.24, ∠PQR = 100°, where P, Q and R are points on a circle with center O. Find ∠OPR.

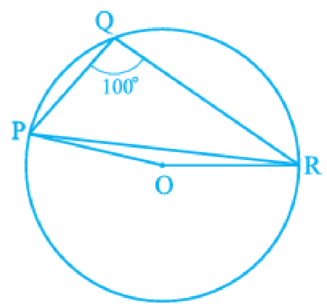


Fig. 9.24

Given

- ∠PQR = 100°
- P, Q, R lie on a circle with center O

To Find : $\angle OPR$

Find the angle at the center

Angle at the center is twice the angle at the circumference on the same arc.

$$\angle POR = 2 \times \angle PQR$$
$$\angle POR = 2 \times 100° = 200°$$

Find the smaller angle at the center

$$\text{Smaller } \angle POR = 360° - 200° = 160°$$

In triangle OPR:

- OP = OR (radii)

So triangle OPR is isosceles.

$$\angle OPR = \angle ORP$$

Sum of angles in triangle:

$$\angle OPR + \angle ORP + \angle POR = 180°$$
$$2\angle OPR + 160° = 180°$$
$$2\angle OPR = 20°$$
$$\angle OPR = 10°$$

Therefore, $\angle OPR = 10°$

Q. 4 In Fig. 9.25, ∠ABC = 69°, ∠ACB = 31°. Find ∠BDC.

Given

- ∠ABC = 69°
- ∠ACB = 31°

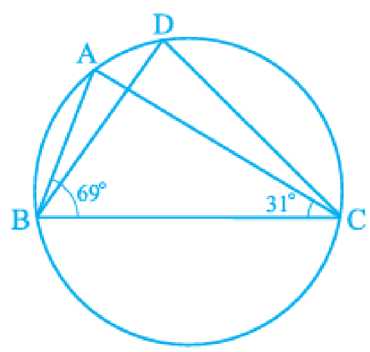


Fig. 9.25

To Find $\angle BDC$

In triangle ABC:

$$\angle BAC = 180° - (69° + 31°)$$
$$\angle BAC = 80°$$

Use the circle theorem

Angles standing on the same chord BC are equal.

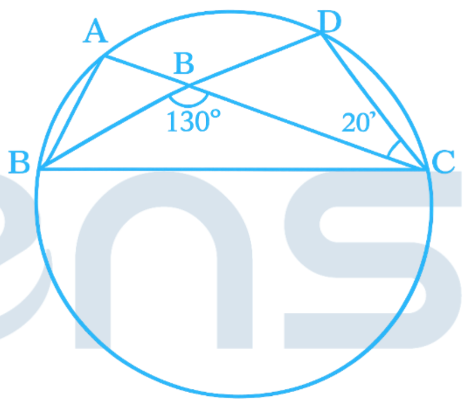
$$\angle BDC = \angle BAC$$

Therefore, $\angle BDC = 80°$

Q.5 In Fig. 9.26, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$. Find $\angle BAC$. (Fig. 9.26)

Given

- A, B, C, D lie on a circle
- AC and BD intersect at E
- $\angle BEC = 130°$
- $\angle ECD = 20°$



To Find $\angle BAC$

$$\angle BEC = \angle AED = 130°$$

In triangle AEC

$$\angle AEC = 180° - 130° = 50°$$

Now, Find $\angle BAC$

Angles in the same segment are equal.

$$\angle BAC = \angle BDC$$

Also,

$$\angle BDC = \angle BEC - \angle ECD$$
$$\angle BDC = 130° - 20° = 110°$$

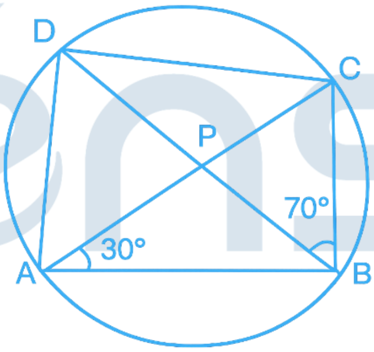
Angle at the circumference is half the angle at the center:

$$\angle BAC = \frac{1}{2} \times 110° = 55°$$

Therefore, $\angle BAC = 55°$

Q. 6 ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70° and ∠BAC = 30°, find ∠BCD. Further, if AB = BC, find ∠ECD.

Given



- ABCD is a cyclic quadrilateral.
- Diagonals AC and BD intersect at E.
- ∠DBC = 70°
- ∠BAC = 30°

To Find

1. ∠BCD
2. If AB = BC, find ∠ECD

Find ∠BCD

Angles standing on the same chord of a circle are equal.

- ∠DBC and ∠DAC stand on chord DC.

Therefore:

$$\angle DAC = \angle DBC = 70°$$

Now consider triangle ABC.

Given:

$$\angle BAC = 30°$$

Angle at C subtended by arc BD is:

$$\angle BCD = \angle BAD$$

But:

$$\angle BAD = \angle DAC + \angle CAB$$

Substitute values:

$$\angle BAD = 70° + 30° = 100°$$

Hence: $\angle BCD = 100°$

Part (ii): To Find ∠ECD

Given AB = BC

If AB = BC, then triangle ABC is isosceles.

So:

$$\angle BCA = \angle CAB = 30°$$

Now:

$$\angle BCD = 100°$$

Thus:

$$\angle ECD = \angle BCD - \angle BCA$$
$$\angle ECD = 100° - 30° = 70°$$

Therefore, $\angle BCD = 100°$ and $\angle ECD = 70°$

Q. 7 If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution:

Given:

Diagonals of a cyclic quadrilateral are diameters of the circle.

To Prove

The quadrilateral is a rectangle.

Proof

- A diameter subtends an angle of 90° at the circumference.
- Since both diagonals are diameters, each interior angle of the quadrilateral is 90°.

Thus:

$$\angle A = \angle B = \angle C = \angle D = 90°$$

A quadrilateral with all angles equal to 90° is a rectangle.

Hence Proved: **The given cyclic quadrilateral is a rectangle**.

Q.8 If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Given

- ABCD is a trapezium.
- AB ∥ CD
- Non-parallel sides AD = BC

To Prove: The trapezium is cyclic.

Proof

In an isosceles trapezium:

- Base angles are equal.

So:

$$\angle A = \angle B$$

Since AB ∥ CD, interior angles on the same side of the transversal satisfy:

$$\angle A + \angle D = 180°$$

Substitute:

$$\angle B + \angle D = 180°$$

Thus, a pair of opposite angles is supplementary.

A quadrilateral whose opposite angles sum to 180° is cyclic.

Hence Proved

The trapezium is cyclic.

Q.9 Two circles intersect at two points B and C. Through B, two-line segments ABD and PBQ are drawn to intersect the circles at A and D, and P and Q respectively (see Fig. 9.27). Prove that ∠ACP = ∠QCD.

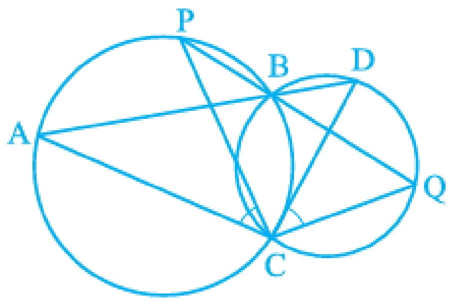


Fig. 9.27

Given

- Two circles intersect at points B and C.
- Through B, line segments ABD and PBQ are drawn.
- They intersect the circles at A, D and P, Q respectively.

To Prove

$$\angle ACP = \angle QCD$$

Proof

- In the left circle, points A, B, C, P lie on the same circle.
- ∠ACP and ∠ABP stand on the same chord AP.

So,

$$\angle ACP = \angle ABP$$

- In the right circle, points D, B, C, Q lie on the same circle.
- ∠QCD and ∠DBQ stand on the same chord DQ.

So,

$$\angle QCD = \angle DBQ$$

- Since AB and DB lie on the same straight line, and PB and QB lie on the same straight line,

$$\angle ABP = \angle DBQ$$

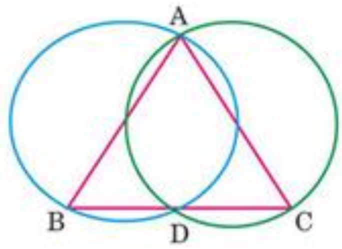
Therefore,

$$\angle ACP = \angle QCD$$

Hence Proved

Q.10 If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lies on the third side.

Given



- In triangle ABC, circles are drawn with AB and AC as diameters.
- The circles intersect at point P.

To Prove : Point D lies on side BC.

Proof

- Angle in a semicircle is 90°.

So,

$$\angle ADB = 90° \text{and} \angle ADC = 90°$$

Thus,

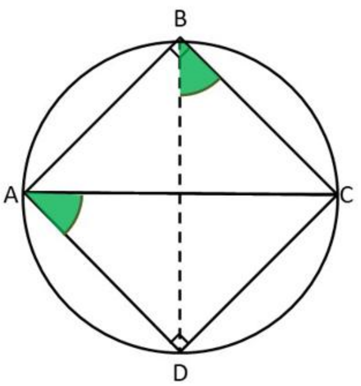
$$\angle BDC = 180°$$

This means B, D, C lie on a straight line.

Hence, D lies on BC.

Hence Proved

Q.11 ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.



Given

- ABC and ADC are two right triangles.
- They have a common hypotenuse AC.

To Prove

$$\angle CAD = \angle CBD$$

Proof

- In right triangle ABC, the right angle is at B.
- In right triangle ADC, the right angle is at D.

Thus, points A, B, C, D lie on a circle with diameter AC.

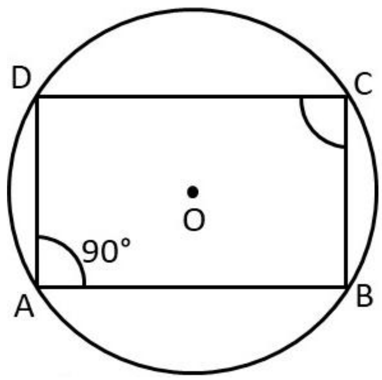
Hence, ABCD is a cyclic quadrilateral.

Angles standing on the same chord CD are equal:

$$\angle CAD = \angle CBD$$

Hence Proved

Q.12 Prove that a cyclic parallelogram is a rectangle.



Given

- A parallelogram ABCD is cyclic.

To Prove

The parallelogram is a rectangle.

Proof

- In a parallelogram:

$$\angle A = \angle C$$

- In a cyclic quadrilateral:

$$\angle A + \angle C = 180°$$

Substitute:

$$\angle A + \angle A = 180°$$
$$2\angle A = 180°$$
$$\angle A = 90°$$

Similarly,

$$\angle B = \angle C = \angle D = 90°$$

Thus, all angles are right angles.

Hence Proved

The cyclic parallelogram is a rectangle.

End of chapter

Exercise 10.1

Q.1 A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Given

- The signal board is an equilateral triangle
- Side length = a
- Perimeter = 180 cm

Find the length of each side

For an equilateral triangle,

$$\text{Perimeter} = 3a$$
$$3a = 180$$
$$a = 60 \text{ cm}$$

Write Heron's Formula

For any triangle with sides a, b, c,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where

$$s = \frac{a+b+c}{2}$$

$$s = \frac{60+60+60}{2} = 90 \text{ cm}$$

Apply Heron's Formula

$$\text{Area} = \sqrt{90(90-60)(90-60)(90-60)}$$
$$= \sqrt{90 \times 30 \times 30 \times 30}$$
$$= \sqrt{2,430,000}$$
$$= \sqrt{243 \times 10,000}$$
$$= 100\sqrt{243}$$
$$= 100 \times 9\sqrt{3}$$
$$= 900\sqrt{3}\ \text{cm}^2$$

therefore, Area of the signal board $= 900\sqrt{3}\ \text{cm}^2$

Q.2 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig. 10.6). The advertisements yield an earning of ₹ 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?

Solution:

Given

The triangular side wall of a flyover has sides:

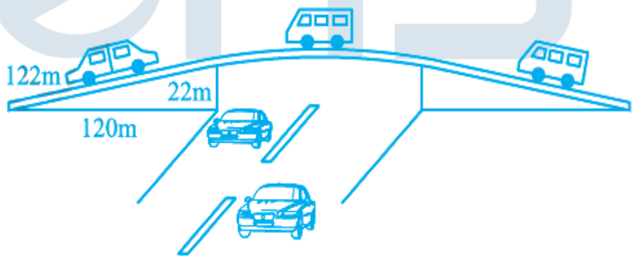


Fig. 12.9

- $a = 122\text{m}$
- $b = 22\text{m}$
- $c = 120\text{m}$

Rate of earning = ₹ 5000 per m^2 per year
Time = 3 months = ¼ year

Step 1: Find the semi-perimeter

$$s = \frac{a+b+c}{2}$$
$$s = \frac{122+22+120}{2} = \frac{264}{2} = 132\ \text{m}$$

Apply Heron's Formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{132(132-122)(132-22)(132-120)}$$
$$= \sqrt{132 \times 10 \times 110 \times 12}$$
$$= \sqrt{1742400}$$
$$= 1320 \text{ m}^2$$

Calculate annual rent

$$\text{Annual rent} = 1320 \times 5000$$
$$= ₹\, 66,00,000$$

Rent for 3 months

$$\text{Rent for 3 months} = \frac{1}{4} \times 66,00,000$$
$$= ₹\, 16,50,000$$

Therefore, Rent paid for 3 months $= ₹\, 16,50,000$

Q.3 There is a slide in a park. One of its side walls has been painted in some color with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig. 10.7). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in color.

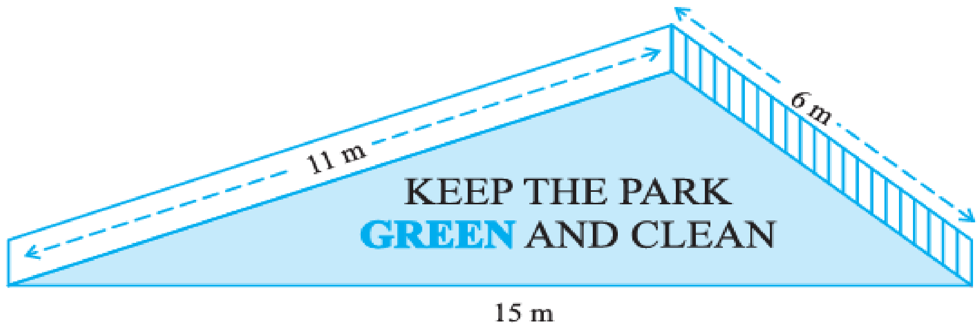


Fig. 10.7

Given

The painted wall is triangular with sides:

- $a = 15\text{m}$
- $b = 11\text{m}$
- $c = 6\text{m}$

Find the semi-perimeter

$$s = \frac{15 + 11 + 6}{2} = \frac{32}{2} = 16 \text{ m}$$

Apply Heron's Formula

$$\begin{aligned}
&\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \\
&= \sqrt{16(16-15)(16-11)(16-6)} \\
&= \sqrt{16 \times 1 \times 5 \times 10} \\
&= \sqrt{800} \\
&= \sqrt{16 \times 50} \\
&= 4\sqrt{50} \\
&= 20\sqrt{2} \text{ m}^2
\end{aligned}$$

Therefore, Area painted $= 20\sqrt{2} \text{ m}^2$

Q.4 Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Given:

Two sides $= 18$ cm and 10 cm
Perimeter $= 42$ cm

$$\text{Third side} = 42 - (18 + 10) = 14 \text{ cm}$$

So, the sides are 18 cm, 10 cm, 14 cm.

Find the semi-perimeter

$$s = \frac{18 + 10 + 14}{2} = \frac{42}{2} = 21 \text{ cm}$$

Apply Heron's Formula

$$\begin{aligned} \text{Area} &= \sqrt{21(21-18)(21-10)(21-14)} \\ &= \sqrt{21 \times 3 \times 11 \times 7} \\ &= \sqrt{4851} \\ &= \sqrt{441 \times 11} \\ &= 21\sqrt{11} \text{ cm}^2 \end{aligned}$$

Therefore, Area= $21\sqrt{11}$ cm^2

Q.5 The sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.

Given: Sides are in the ratio 12: 17: 25
Perimeter = 540 cm

Let each side be in ratio of x

$$12x + 17x + 25x = 54x$$

$$\text{or} \quad \text{x} = \frac{540}{54} = 10 \text{ cm}$$

therefore

$$\begin{aligned} a &= 12 \times 10 = 120 \text{ cm} \\ b &= 17 \times 10 = 170 \text{ cm} \\ c &= 25 \times 10 = 250 \text{ cm} \end{aligned}$$

or

$$s = \frac{120 + 170 + 250}{2} = \frac{540}{2} = 270 \text{ cm}$$

Apply Heron's Formula

$$\begin{aligned} \text{Area} &= \sqrt{270(270-120)(270-170)(270-250)} \\ &= \sqrt{270 \times 150 \times 100 \times 20} \\ &= \sqrt{81{,}000{,}000} \\ &= 9000 \text{ cm}^2 \end{aligned}$$

Therefore, Area $= 9000 \text{ cm}^2$

Q. 6 An isosceles triangle has a perimeter of 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Perimeter $= 30$ cm
Each equal side $= 12$ cm
(Isosceles triangle)

Find the base $= 30 - (12 + 12) = 6$ cm

Find the semi-perimeter

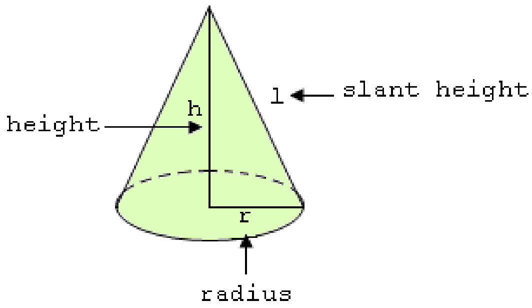
$$s = \frac{12 + 12 + 6}{2} = \frac{30}{2} = 15 \text{ cm}$$

Apply Heron's Formula

$$\begin{aligned} \text{Area} &= \sqrt{15(15-12)(15-12)(15-6)} \\ &= \sqrt{15 \times 3 \times 3 \times 9} \\ &= \sqrt{1215} \\ &= \sqrt{81 \times 15} \\ &= 9\sqrt{15} \text{ cm}^2 \end{aligned}$$

Therefore, Area $= 9\sqrt{15} \text{ cm}^2$

End of Chapter



Exercise 11.1

Q.1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Given:

Diameter = 10.5 cm,
therefore, r = 5.25 cm

Slant height,
l = 10 cm

We know, Curved Surface Area of a cone

CSA = $\pi \times r \times l$

Substitute the values

CSA = $(22/7) \times 5.25 \times 10 = 165 \text{ cm}^2$

Or Curved surface area = 165 cm^2

Q.2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Given: Diameter = 24 m

Radius, r = 12 m

Slant height, l = 21 m

We know, Total Surface Area of a cone TSA = $\pi \times r \times (r + l)$

Putting the values, TSA = $(22/7) \times 12 \times (12 + 21)$

TSA = $(22/7) \times 12 \times 33$

Or TSA = $(22/7) \times 396 = 1244.57\ m^2$ (approx.)

Therefore, Total surface area = $1244.57\ m^2$

Q.3. Curved surface area of a cone is $308\ cm^2$ and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.

Solution:

Given

Curved surface area (CSA) = $308\ cm^2$
Slant height (l) = 14 cm

Find:
(i) radius of the base
(ii) total surface area (TSA)

We know, Curved Surface Area of cone " CSA " = $\pi r l$

Substitute given values

$308 = (22/7) \times r \times 14$

$308 = 44r$

$r = 308 \div 44$

$r = 7$ cm

Formula for Total Surface Area

$TSA = \pi r (r + l)$

Or $TSA = (22/7) \times 7 \times (7 + 14)$
$TSA = 22 \times 21$

$TSA = 462 \text{ cm}^2$

Final Answers

(i) Radius = 7 cm
(ii) Total surface area = 462 cm^2

Q.4. A conical tent is 10 m high and the radius of its base is 24 m. Find (i) slant height of the tent. (ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹ 70.

Given:

Height (h) = 10 m
Radius (r) = 24 m

Find:
(i) slant height
(ii) cost of canvas (₹70 per m^2)

We know, Formula for slant height

$l = \sqrt{(r^2 + h^2)}$

Substitute values

$l = \sqrt{(24^2 + 10^2)}$
$l = \sqrt{(576 + 100)}$
$l = \sqrt{676}$

$l = 26$ m

Curved surface area of tent

Curved Surface Area = $\pi r l$

CSA = $(22/7) \times 24 \times 26$

CSA = 1961.14 m^2

Cost of canvas

Cost of 1m^2 of canvas = ₹ 70

Cost = 1961.14×70

Cost = ₹ 1,37,280 (approx.)

Therefore (i) Slant height = 26 m
(ii) Cost of canvas = ₹ 1,37,280

Q.5. What length of tarpaulin 3 m wide will be required to make a conical tent of height 8 m and base radius 6 m? Assume that the extra length of material required for stitching margins and wastage in cutting is approximately 20 cm. (Use $\pi = 3.14$).

Given:

Width of tarpaulin = 3 m
Height of tent = 8 m

Radius = 6 m
Extra material = 20%

Slant height

$l = \sqrt{(6^2 + 8^2)}$
$l = \sqrt{(36 + 64)}$
$l = \sqrt{100}$

$l = 10$ m

Curved surface area

CSA = $\pi r l$

CSA = $3.14 \times 6 \times 10$
CSA = $188.4\ m^2$

Add 20% wastage

Extra area = 20% of 188.4
= 37.68

Total area = $188.4 + 37.68 = 226.08\ m^2$

Length of tarpaulin

Length = Area ÷ Width

Length = $226.08 \div 3$

Length = 75.36 m

Therefore, Required length of tarpaulin = 75.36 m

Q.6 The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹ 210 per 100 m^2.

Given:

Slant height = 25 m
Base diameter = 14 m → radius = 7 m
Rate = ₹210 per 100 m^2

Curved surface area

CSA = π r l

CSA = (22/7) × 7 × 25
CSA = 550 m^2

Cost calculation

Cost for 100 m^2 = ₹210

Cost for 550 m^2 = (550 ÷ 100) × 210

Cost = ₹ 1155

Therefore, Cost of white-washing = ₹ 1155

Q.7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Given:

Radius = 7 cm
Height = 24 cm
Number of caps = 10

Slant height

$l = \sqrt{(7^2 + 24^2)}$
$l = \sqrt{(49 + 576)}$
$l = \sqrt{625}$

$l = 25$ cm

CSA of one cap

CSA = π r l

CSA = $(22/7) \times 7 \times 25$
CSA = 550 cm^2

Area for 10 caps

Total area = 10×550

Total area = 5500 cm^2

Therefore, Area of sheet required = 5500 cm^2

Q.8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$).

Solution:

Given:

Number of cones = 50

Diameter = 40 cm → radius = 20 cm

Height = 1 m = 100 cm

Painting rate = ₹12 per m^2

Slant height

$l = \sqrt{(20^2 + 100^2)}$

$l = \sqrt{(400 + 10000)}$

$l = \sqrt{10400}$

$l = 100 \times \sqrt{1.04}$

$l = 102$ cm

CSA of one cone $= \pi r l$

CSA = $3.14 \times 20 \times 102$

CSA = 6405.6 cm^2

Convert to m^2

6405.6 cm^2 = 0.64056 m^2

Area of 50 cones

Total area = $50 \times 0.64056 = 32.028$ m^2

Cost of painting @ ₹ 12 per square meter

Cost = 32.028×12 = ₹ 384.34 (approx.)

Therefore, Cost of painting all cones = ₹ 384.34

End of Exercise

Exercise 11.2

Q.1. Find the surface area of a sphere of radius:

Given Formula

Surface Area of a sphere $= 4\pi r^2$

(i) Radius $= 10.5$ cm

Given : $r = 10.5$ cm
Surface Area of a sphere $= 4\pi r^2$

Substitute in the formula

$$\text{Surface Area} = 4 \times \frac{22}{7} \times (10.5)^2$$

$$\text{Surface Area} = 4 \times \frac{22}{7} \times 110.25$$

$$\text{Surface Area} = 1386 \text{ cm}^2$$

Therefore, Surface area $= 1386 \text{ cm}^2$

(ii) Radius (r) $= 5.6$ cm

Substitute in the formula

$$\text{Surface Area} = 4 \times \frac{22}{7} \times (5.6)^2$$

$$\text{Surface Area} = 4 \times \frac{22}{7} \times 31.36$$

$$\text{Surface Area} = 394.24 \text{ cm}^2$$

Surface area $= 394.24 \text{ cm}^2$

(iii) Radius = 14 cm

$$r = 14 \text{ cm}$$

Substitute in the formula

$$\text{Surface Area} = 4 \times \frac{22}{7} \times (14)^2$$
$$\text{Surface Area} = 4 \times \frac{22}{7} \times 196$$
$$\text{Surface Area} = 2464 \text{ cm}^2$$

Therefore, Surface area = 2464 cm^2

Q.2. Find the surface area of a sphere of diameter:
(i) 14 cm
(ii) 21 cm
(iii) 3.5 m

Formula:

$$\text{Surface Area} = 4\pi r^2 \text{and} r = \frac{d}{2}$$

(i) Diameter = 14 cm

$$r = 7 \text{ cm}$$
$$\text{Surface Area} = 4 \times \frac{22}{7} \times 7^2 = 616 \text{ cm}^2$$

(ii) Diameter = 21 cm

$$r = 10.5 \text{ cm}$$
$$\text{Surface Area} = 4 \times \frac{22}{7} \times (10.5)^2 = 1386 \text{ cm}^2$$

(iii) Diameter = 3.5 m

$$r = 1.75 \text{ m}$$

$$\text{Surface Area} = 4 \times \frac{22}{7} \times (1.75)^2 = 38.5 \text{ m}^2$$

Q.3 Find the total surface area of a hemisphere of radius 10 cm. (Use π = 3.14).

Given:

r = 10 cm

$$\text{Total Surface Area of hemisphere} = 3\pi r^2$$
$$= 3 \times 3.14 \times 10^2 = 942 \text{ cm}^2$$

Answer: 942 cm^2

Q. 4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Ratio of surface areas when radius increases from 7 cm to 14 cm

$$\text{SA} \propto r^2$$
$$\Rightarrow 7^2 : 14^2 = 49 : 196 = 1 : 4$$

Answer: 1: 4

Q. 5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹ 16 per 100 cm^2.

Given:

inner diameter = 10.5 cm

therefore, $r = 5.25$ cm

Inside area = Curved surface area of hemisphere

$$\text{CSA} = 2\pi r^2 = 2 \times \frac{22}{7} \times (5.25)^2 = 173.25 \text{ cm}^2$$

Cost @ ₹16 per 100 cm^2

$$\text{Cost} = \frac{173.25}{100} \times 16 = ₹\,27.72$$

Answer: ₹ 27.72

Q.6. Find the radius of a sphere whose surface area is 154 cm^2.

Radius of a sphere with surface area = 154 cm^2

$$4\pi r^2 = 154$$
$$4 \times \frac{22}{7} \times r^2 = 154$$
$$r^2 = \frac{154 \times 7}{88} = 12.25$$
$$r = 3.5 \text{ cm}$$

Answer: $r = 3.5$ cm

Q.7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Solution:

Ratio of surface areas: Moon : Earth

Given diameter of Moon = ¼ diameter of Earth

$$r_{\text{moon}} : r_{\text{earth}} = 1 : 4$$

$$SA \propto r^2 \Rightarrow 1^2 : 4^2 = 1 : 16$$

Answer: 1 : 16

Q. 8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Solution:

Thickness = 0.25 cm
Inner radius = 5 cm

$$\text{Outer radius} = 5 + 0.25 = 5.25 \text{ cm}$$

Formula (outer CSA of hemisphere):

$$CSA = 2\pi R^2$$

$$= 2 \times \frac{22}{7} \times (5.25)^2 = 173.25 \text{ cm}^2$$

Answer : 173.25 cm^2

Q. 9. A right circular cylinder just encloses a sphere of radius r (see Fig. 11.10). Find:
(i) surface area of the sphere,
(ii) curved surface area of the cylinder,
(iii) ratio of the areas obtained in (i) and (ii).

Sphere just enclosed by a right circular cylinder (radius r)

Facts:

- Radius of cylinder = r
- Height of cylinder = 2r

(i) Surface area of sphere $= 4\pi r^2$

(ii) Curved surface area of cylinder $= 2\pi rh = 2\pi r(2r) = 4\pi r^2$

(iii) Ratio of (i) and (ii) $4\pi r^2 : 4\pi r^2 = 1:1$

End of Exercise

Exercise 11.3

Useful Formulae

- Volume of a cone:

$$V = \frac{1}{3}\pi r^2 h$$

- 1 litre = 1000 cm^3
- 1 kilolitre = 1000 litres

Q. 1. Find the volume of the right circular cone with:
(i) radius 6 cm, height 7 cm
(ii) radius 3.5 cm, height 12 cm

(i) $r = 6\text{cm}, h = 7\text{cm}$

$$V = \frac{1}{3} \times \frac{22}{7} \times 6^2 \times 7 = \frac{1}{3} \times 22 \times 36 = 264 \text{ cm}^3$$

(ii) $r = 3.5\text{cm}, h = 12\text{cm}$

$$V = \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12 = \frac{1}{3} \times \frac{22}{7} \times 12.25 \times 12 = 154 \text{ cm}^3$$

Q. 2. Find the capacity in litres of a conical vessel with:
(i) radius 7 cm, slant height 25 cm
(ii) height 12 cm, slant height 13 cm

Solution:

Capacity of a conical vessel

(i) $r = 7\text{cm}, h = 25\text{cm}$

$$V = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 25 = \frac{3850}{3} = 1283.33\ \text{cm}^3$$
$$\text{Capacity} = 1.283\ \text{litres}$$

(ii) $h = 12\text{cm}$, slant height $l = 13\text{cm}$

$$r = \sqrt{l^2 - h^2} = \sqrt{13^2 - 12^2} = \sqrt{25} = 5\ \text{cm}$$
$$V = \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 12 = 314.29\ \text{cm}^3 = 0.314\ \text{litres}$$

Q. 3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use $\pi = 3.14$).

Solution:

Height $h = 15\text{cm}$, volume $= 1570\text{cm}^3$ (use $\pi = 3.14$)

$$1570 = \frac{1}{3} \times 3.14 \times r^2 \times 15 \Rightarrow 1570 = 15.7r^2$$
$$r^2 = 100 \Rightarrow r = 10\ \text{cm}$$

Q. 4. If the volume of a right circular cone of height 9 cm is $48\pi\ \text{cm}^3$, find the diameter of its base.

Solution:

Volume $= 48\pi\text{cm}^3$, height $= 9\text{cm}$

$$48\pi = \frac{1}{3}\pi r^2 \times 9 \Rightarrow 48 = 3r^2 \Rightarrow r^2 = 16 \Rightarrow r = 4\ \text{cm}$$

Therefore, Diameter $= 8$ cm

Q. 5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kiloliters?

Solution:

Conical pit (top diameter = 3.5m, depth = 12m)

$$r = 1.75 \text{ m}$$

$$V = \frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12 = 38.5 \text{ m}^3$$
$$= 38.5 \text{ kilolitres}$$

Therefore capacity = 38.5 kilolitres

Q. 6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find:
(i) height of the cone
(ii) slant height of the cone
(iii) curved surface area of the cone

Solution:

Volume = 9856cm^3, diameter = 28cm

$$r = 14 \text{ cm}$$

(i) Height

$$9856 = \frac{1}{3} \times \frac{22}{7} \times 14^2 \times h \Rightarrow h = 48 \text{ cm}$$

(ii) Slant height

$$l = \sqrt{r^2 + h^2} = \sqrt{14^2 + 48^2} = \sqrt{2500} = 50 \text{ cm}$$

(iii) Curved surface area

$$\text{CSA} = \pi r l = \frac{22}{7} \times 14 \times 50 = 2200 \text{ cm}^2$$

Q. 7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Triangle with sides 5 cm, 12 cm, 13 cm revolved about side 12 cm

Here, radius = 5 cm, height = 12 cm

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 12 = 314.29 \text{ cm}^3$$

Q. 8. If the triangle ABC in Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Solution:

Same triangle revolved about side 5 cm

Now, radius = 12 cm, height = 5 cm

$$V = \frac{1}{3} \times \frac{22}{7} \times 12^2 \times 5 = 754.29 \text{ cm}^3$$

Ratio of volumes (Q7 : Q8)

$$314.29 : 754.29 = 25 : 60 = 5 : 12$$

Q.9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Heap of wheat (diameter = 10.5m, height = 3m)

$$r = 5.25 \text{ m}$$

Volume

$$V = \frac{1}{3} \times \frac{22}{7} \times (5.25)^2 \times 3 = 86.625 \text{ m}^3$$

Area of canvas required (CSA)

$$\text{CSA} = \pi r l$$

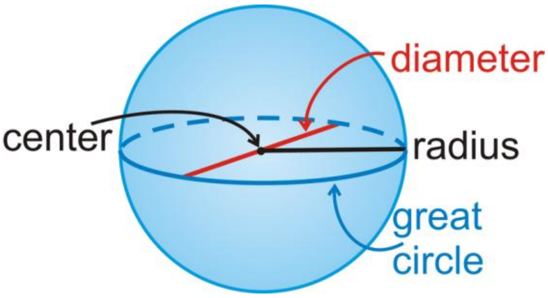
$$l = \sqrt{r^2 + h^2} = \sqrt{(5.25)^2 + 3^2} = 6 \text{ m}$$

$$\text{CSA} = \frac{22}{7} \times 5.25 \times 6 = 99 \text{ m}^2$$

End of Exercise

Exercise 11.4

Reference Diagrams (Sphere & Hemisphere)



Useful Formulae

- Volume of a sphere: $V = \frac{4}{3}\pi r^3$
- Volume of a hemisphere: $V = \frac{2}{3}\pi r^3$
- Surface area of a sphere: $S = 4\pi r^2$
- 1 litre = 1000 cm^3

Q.1. Find the volume of a sphere whose radius is:
(i) 7 cm
(ii) 0.63 m

(i) Radius $r = 7$cm

$$V = \frac{4}{3} \times \frac{22}{7} \times 7^3 = \frac{4}{3} \times 22 \times 49 = \frac{4312}{3} = 1437.33 \text{ cm}^3$$

(ii) Radius $r = 0.63$m

$$V = \frac{4}{3} \times \frac{22}{7} \times (0.63)^3 = 1.0476 \text{ m}^3 \text{ (approx.)}$$

Q. 2. Find the amount of water displaced by a solid spherical ball of diameter:
(i) 28 cm
(ii) 0.21 m

(i) Diameter = 28 cm

$$r = 14 \text{ cm}$$
$$V = \frac{4}{3} \times \frac{22}{7} \times 14^3 = \frac{4}{3} \times 22 \times 392 = 11498.67 \text{ cm}^3$$

(ii) Diameter = 0.21 m

$$r = 0.105 \text{ m}$$
$$V = \frac{4}{3} \times \frac{22}{7} \times (0.105)^3 = 0.00485 \text{ m}^3$$

Q. 3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3?

Solution:

Diameter = 4.2 cm ⇒ radius = 2.1 cm
Density = 8.9 g/cm^3

$$V = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 = 38.808 \text{ cm}^3$$
$$\text{Mass} = \text{Density} \times \text{Volume} = 8.9 \times 38.808 = 345.39 \text{ g}$$

Q.4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Solution:

Moon–Earth volume fraction

Given diameter of moon = ¼ diameter of earth

$$\text{Volume} \propto r^3 \Rightarrow \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

Answer: The moon's volume is $\frac{1}{64}$of the earth's volume.

Q. 5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

Solution:

Capacity of a hemispherical bowl

Diameter = 10.5 cm ⇒ radius = 5.25 cm

$$V = \frac{2}{3} \times \frac{22}{7} \times (5.25)^3 = 303.19 \text{ cm}^3$$

$$\text{Capacity} = 0.303 \text{ litres}$$

therefore, capacity of hemispherical bowl = 0.303 litres

Q. 6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Solution:

Thickness = 1 cm
Inner radius = 1 m ⇒ outer radius = 1.01 m

$$\text{Volume of iron} = \frac{2}{3}\pi(R^3 - r^3)$$

$$= \frac{2}{3} \times \frac{22}{7}(1.01^3 - 1^3) = 0.0627 \text{ m}^3$$

therefore, volume of iron used : 0.0627 m^3

Q. 7. Find the volume of a sphere whose surface area is 154 cm^2.

Volume of a sphere with surface area = 154 cm^2

$$4\pi r^2 = 154 \Rightarrow r^2 = \frac{154 \times 7}{88} = 12.25 \Rightarrow r = 3.5 \text{ cm}$$
$$V = \frac{4}{3} \times \frac{22}{7} \times (3.5)^3 = 179.67 \text{ cm}^3$$

therefore, volume of sphere : 179.67 cm^3

Q.8. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹ 4989.60. If the cost of white-washing is ₹ 20 per square meter, find:
(i) inner surface area of the dome
(ii) volume of the air inside the dome.

Solution:

Dome in the form of a hemisphere

Cost = ₹4989.60
Rate = ₹20 per m^2

(i) Inside surface area

$$\text{Area} = \frac{4989.60}{20} = 249.48 \text{ m}^2$$

(ii) Volume of air inside the dome

$$2\pi r^2 = 249.48 \Rightarrow r^2 = 39.7 \Rightarrow r = 6.3 \text{ m}$$
$$V = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (6.3)^3 = 523.9 \text{ m}^3$$

Q.9. Twenty-seven solid iron spheres, each of radius r and surface area S, are melted to form a sphere with surface area S'. Find:
(i) radius r' of the new sphere
(ii) ratio of S and S'.

Let radius of each small sphere = r

(i) Radius of new sphere

$$27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r'^3 \Rightarrow r'^3 = 27r^3 \Rightarrow r' = 3r$$

(ii) Ratio of surface areas

$$S: S' = 4\pi r^2: 4\pi(3r)^2 = 1: 9$$

Q. 10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Solution:

Volume of medicine in a spherical capsule

Diameter = 3.5 mm ⇒ radius = 1.75 mm

$$V = \frac{4}{3} \times \frac{22}{7} \times (1.75)^3 = 22.46 \text{ mm}^3$$

Therefore, volume of medicine needed = 22.46 mm^3

End of Chapter

Exercise 12.1

Q.1 A survey conducted by an organization for the cause of illness and death among the women between the ages 15–44 years worldwide, found the following figures (in %):

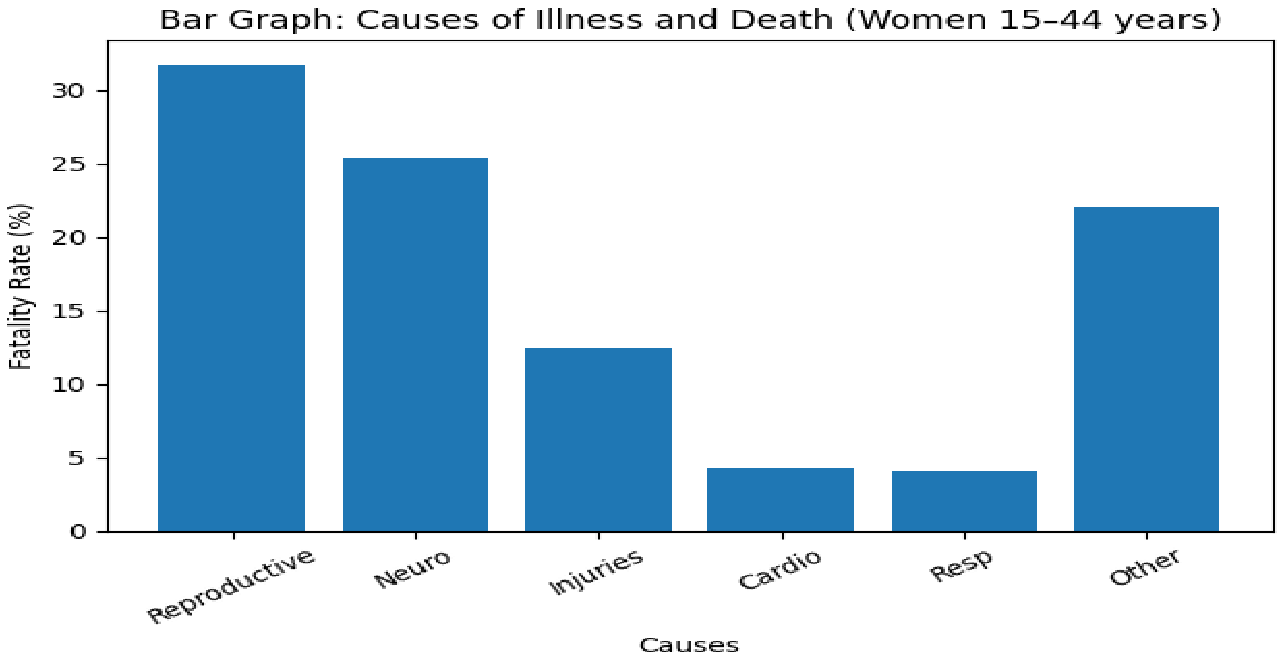
S.No.	Causes	Female fatality rate (%)
1	Reproductive health conditions	31.8
2	Neuropsychiatric conditions	25.4
3	Injuries	12.4
4	Cardiovascular conditions	4.3
5	Respiratory conditions	4.1
6	Other causes	22.0

(i) Represent the information given above graphically.

(ii) Which condition is the major cause of women's ill health and death worldwide?

(iii) Try to find out any two factors responsible for the major cause.

Solution (i) The data is represented using a bar graph shown below:



(ii) The major cause is Reproductive health conditions (31.8%).

(iii) Two factors are:
1. Lack of proper healthcare facilities.
2. Lack of awareness and education.

Q.2 The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below:

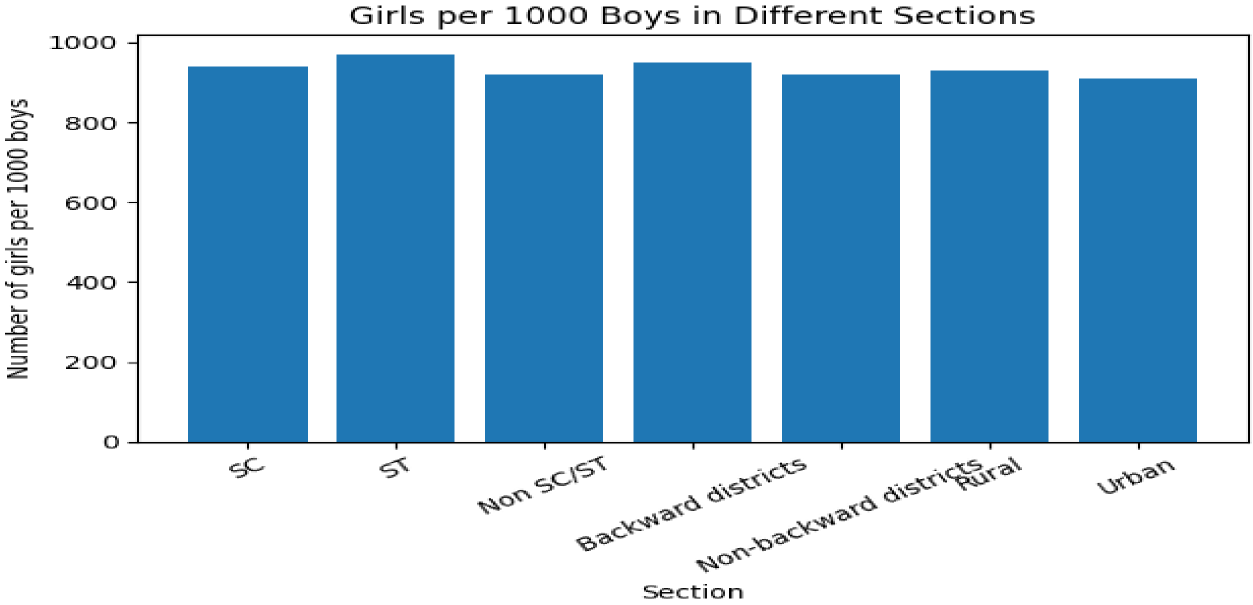
Section	Number of girls per thousand boys
Scheduled Caste (SC)	940
Scheduled Tribe (ST)	970
Non SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.

(ii) In the classroom discuss what conclusions can be arrived at from the graph.

Solution

(i) The bar graph representing the data is shown below:



(ii) Conclusions:

1. Scheduled Tribe (ST) has the highest number of girls (970).
2. Urban areas have the lowest number of girls (910).
3. Backward districts have more girls than non-backward districts.
4. Rural areas have slightly more girls than urban areas.
5. Overall, the number of girls is less than boys in all sections.

Q.3 Given below are the seats won by different political parties in the polling outcome of a state assembly election:

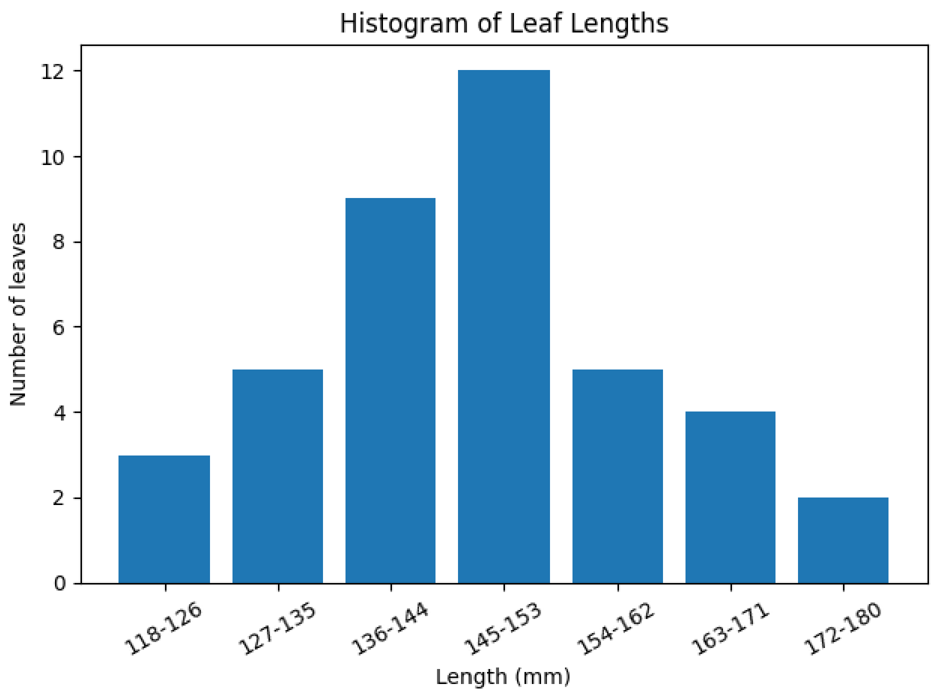
Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

(i) Draw a bar graph to represent the polling results.

(ii) Which political party won the maximum number of seats?

Solution

(i) The histogram is shown below:



(ii) Yes, a frequency polygon can also be used.

(iii) No, it is not correct because the class interval 145–153 contains 12 leaves, but we do not know exact individual lengths. So we cannot say all are exactly 153 mm.

Q.4 The lengths of 40 leaves of a plant were measured and represented in grouped form.

Length (in mm)	Number of Leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

(i) Draw a histogram to represent the given data.

[Hint: First make the class intervals continuous]

(ii) Is there any other suitable graphical representation for the same data?

(iii) Is it correct to conclude that the maximum number of leaves are 153 mm long? Why?

Solution

The lengths of 40 leaves are given below:

Length (in mm)	Number of Leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

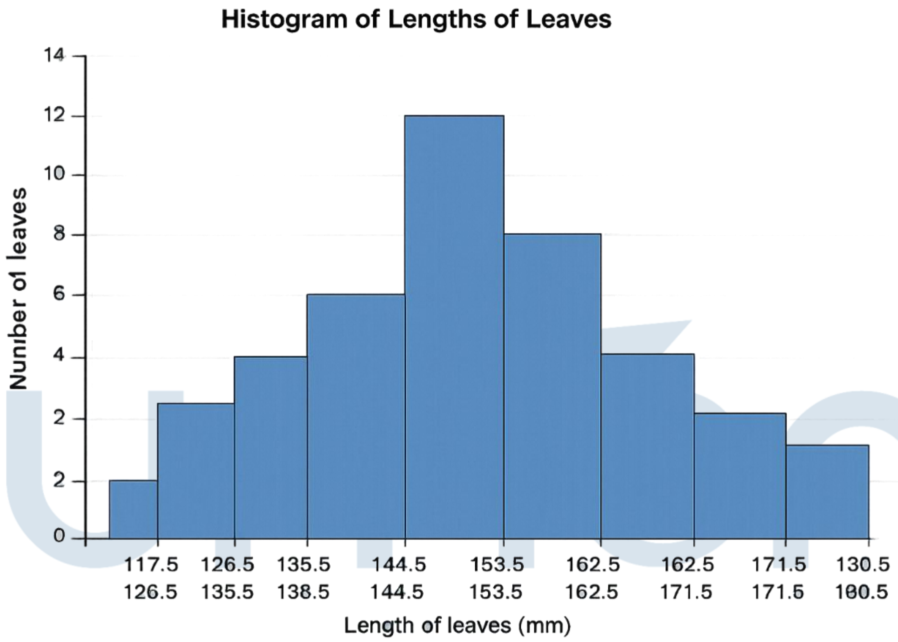
Step 1: Convert into Continuous Class Intervals

Since the measurements are correct to the nearest millimetre, we make the intervals continuous by subtracting 0.5 from each lower limit and adding 0.5 to each upper limit.

Original Interval	Continuous Interval	Frequency
118 – 126	117.5 – 126.5	3
127 – 135	126.5 – 135.5	5
136 – 144	135.5 – 144.5	9
145 – 153	144.5 – 153.5	12
154 – 162	153.5 – 162.5	5
163 – 171	162.5 – 171.5	4
172 – 180	171.5 – 180.5	2

(i) Histogram

- X-axis → Length of leaves (mm)
- Y-axis → Number of leaves
- Draw adjoining rectangles for each interval.



(ii) Another Suitable Graphical Representation

Yes, a Frequency Polygon is also suitable for representing the data.

(iii) Is it Correct to Conclude that the Maximum Number of Leaves are 153 mm Long?

No, this conclusion is incorrect.

The class interval 145–153 mm has the highest frequency (12), but this only means:

12 leaves have lengths between 145 mm and 153 mm.

It does not mean all 12 leaves are exactly 153 mm long.

Correct Conclusion

The maximum number of leaves lie in the interval **145 mm to 153 mm**.

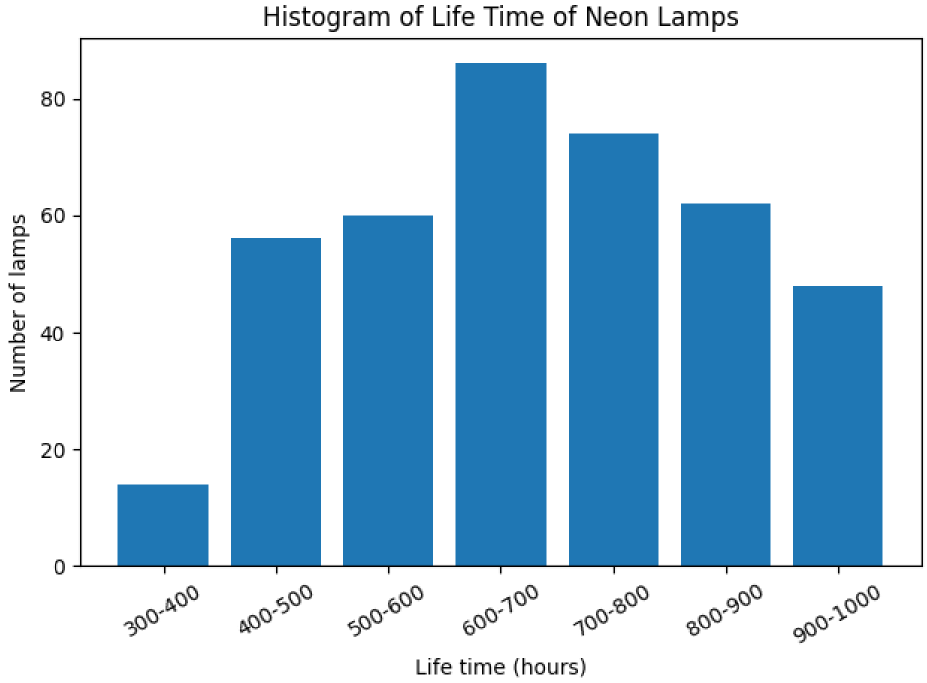
Q.5 The following table gives the life times of 400 neon lamps:

Life time (in hours)	Number of lamps
300-400	14
400-500	56
500-600	60
600-700	86
700-800	74
800-900	62
900-1000	48

(i) Represent the given information with the help of a histogram.

(ii) How many lamps have a life time of more than 700 hours?
Solution

Solution: (i) Histogram is given below:



(ii) Lamps with life more than 700 hours = 74 + 62 + 48 = 184 lamps.

Q.6 The following table gives the distribution of students of two sections according to the marks obtained by them:

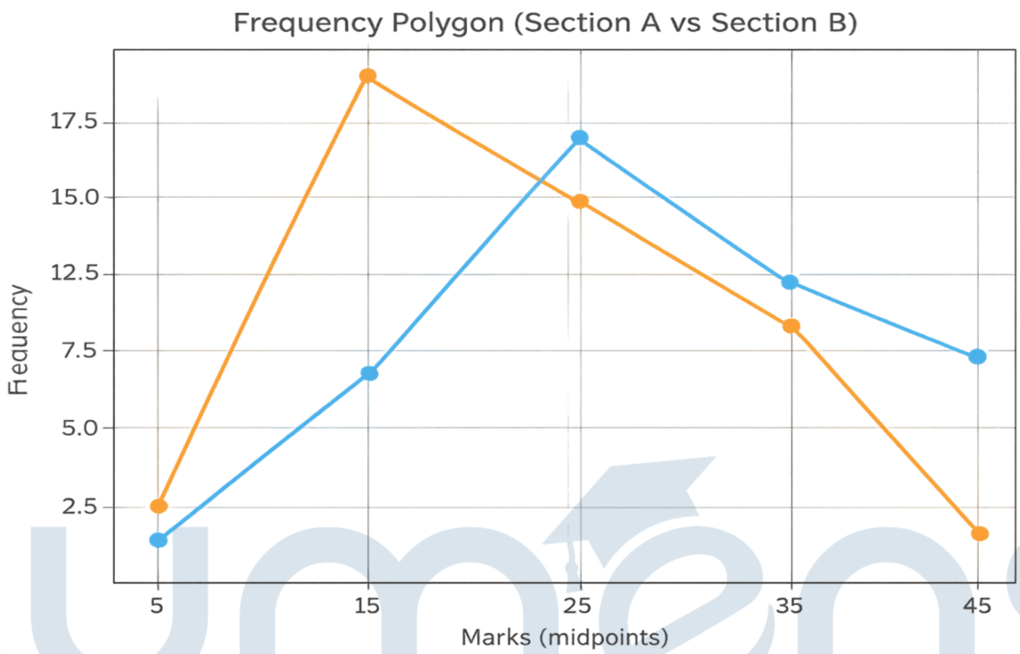
Section A Marks	Frequency	Section B Marks	Frequency
0-10	3	0-10	5
10-20	9	10-20	19
20-30	17	20-30	15
30-40	12	30-40	10
40-50	9	40-50	1

Represent the marks of both sections on the same graph by two frequency polygons. Compare their performance.

Solution:

Class midpoints are taken as: 5, 15, 25, 35, 45.

Frequency polygon is shown below:



Comparison:
1. Section A has highest frequency at 20–30 (17 students).
2. Section B has highest frequency at 10–20 (19 students).
3. Section A performs better in higher marks range (30–50).
4. Section B has more students in lower range.
5. Overall, Section A shows better performance.

Q.7 The runs scored by two teams A and B on the first 60 balls in a cricket match are given below:

Number of balls	Team A	Team B
1-6	2	5
7-12	1	6
13-18	8	2
19-24	9	10
25-30	4	5
31-36	5	6
37-42	6	3
43-48	10	4
49-54	6	8
55-60	2	10

Represent the data of both the teams on the same graph by frequency polygons. [Hint: First make the class intervals continuous.]

Solution

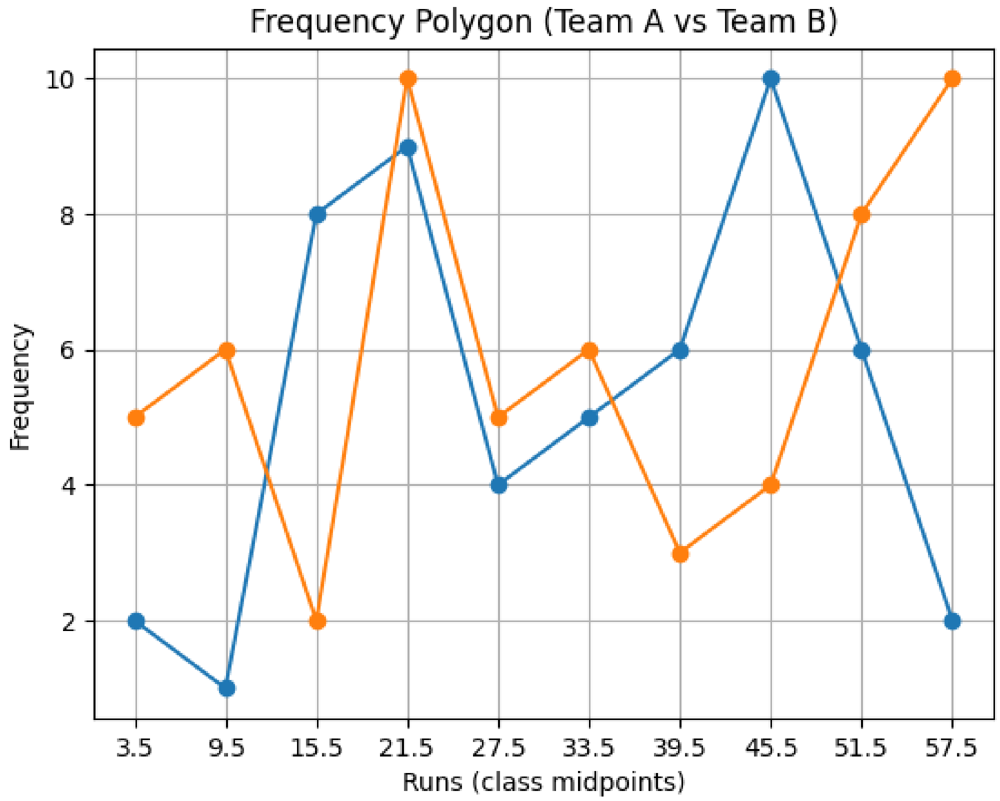
First, convert class intervals into continuous form:

0.5–6.5, 6.5–12.5, 12.5–18.5, 18.5–24.5, 24.5–30.5, 30.5–36.5, 36.5–42.5, 42.5–48.5, 48.5–54.5, 54.5–60.5

Class midpoints are:

3.5, 9.5, 15.5, 21.5, 27.5, 33.5, 39.5, 45.5, 51.5, 57.5

Frequency polygon is shown below:



Comparison:
1. Team B scored more frequently in lower intervals.
2. Team A performed better in middle intervals.
3. Team B shows higher variation.
4. Overall, Team A shows more consistent performance.

Q.8 A random survey of the number of children of various age groups playing in a park was found as follows

Age (in years)	Number of children
1-2	5
2-3	3
3-5	6
5-7	12
7-10	9
10-15	10
15-17	4

Draw a histogram to represent the data above.

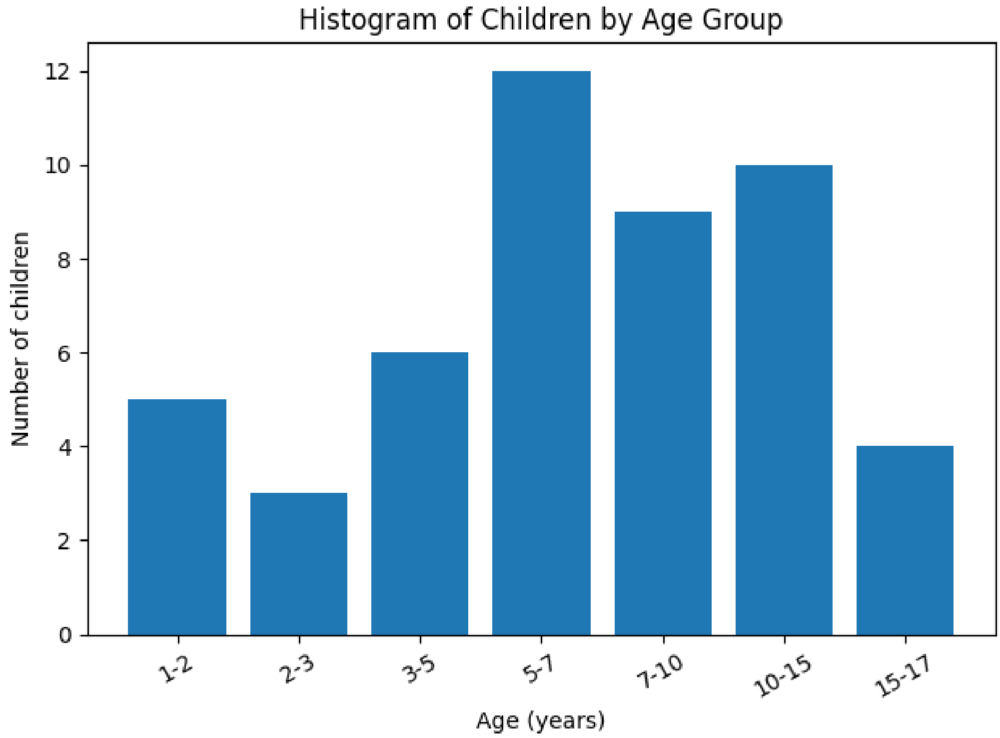
Solution

Step 1: Convert class intervals into continuous form:

0.5–2.5, 2.5–3.5, 3.5–5.5, 5.5–7.5, 7.5–10.5, 10.5–15.5, 15.5–17.5

Step 2: Draw histogram using given frequencies.

------Histogram on next page------



Q.9. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:

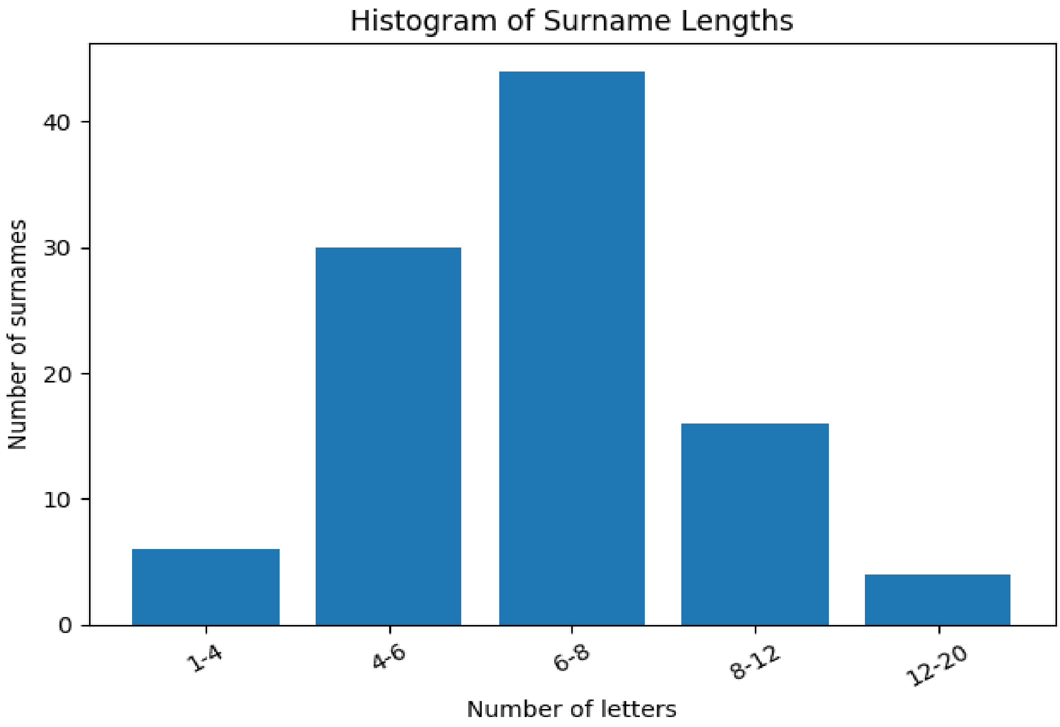
Number of letters	Number of surnames
1-4	6
4-6	30
6-8	44
8-12	16
12-20	4

(i) Draw a histogram to depict the given information.

(ii) Write the class interval in which the maximum number of surnames lie.

Solution:

(i) Histogram is shown below:



(ii) The maximum number of surnames lie in the class interval 6-8.

End of Chapter